Mathematical Modeling on Network Flow Using Fuzzy Cardinality Sets

VANITHA MUTHU P
Associate Professor and Head,
Department of Computer Science and Engineering,
Government College of Engineering Srirangam, Tiruchirappalli – 620012
(Affiliated to Anna University Chennai), Tamil Nadu, India

Abstract

The method, which is based on the application of fuzzy cardinality sets, should be used in the analysis of the flow in the network, according to the suggestions made in this paper. Many times, uncertainty and imprecision involved in the real-world systems cannot be effectively captured by tradition network flow models. Therefore, instead of using membership degree of occurrence in the set of flows, I use the fuzzy set theory, particularly, the fuzzy cardinality set to develop a less sensitive and more general mathematical model for flow problems in network. This allows the proposed models to replicate more accurate forecasts of flow capacities, demands as well as costs. This paper also outlines theory for such solution, the techniques of solving fuzzy network flow problem and the demonstration of the solution through numerical example. Thus, the analysis carried out in the present study showed that there is an improvement in the features of decision making using the proposed fuzzy cardinality set in comparison to the deterministic one in an uncertain environment. This research enriches the literature in the area of fuzzy optimization and provides solution to the problems encountered in network flow to the practitioners particularly in areas of transport, communication and supply chain management.

Keywords: Capacity, Flow, Cardinality, Network, Set

I. INTRODUCTION

Mathematical modeling of network flow using fuzzy cardinality sets has been explored in various contexts. Ghatee & Hashemi (2009) proposed a fuzzy multicommodity flow problem framework, addressing supply, demand, and cost uncertainties using linguistic variables and trapezoidal fuzzy numbers. Bozhenyuk et al. (2012) developed methods for determining maximum flow and minimum cost flow in fuzzy networks with triangular fuzzy numbers, proposing a novel technique for addition and subtraction to maintain self-descriptiveness. Mathew & Mordeson (2017) introduced directed fuzzy networks as normalized node capacitated models, providing fuzzy versions of Menger's theorem and the Max flow Min cut theorem. Earlier, Chanas & Kołodziejczyk (1984) generalized the Ford-Fulkerson theorem for networks with fuzzy capacity constraints and developed an algorithm for determining optimal real-valued flows.

These approaches collectively demonstrate the versatility of fuzzy set theory in modeling complex network flow problems under uncertainty. These papers explore the application of fuzzy set theory to network flow problems. Chanas and Kołodziejczyk (1982, 1986) introduced fuzzy capacity constraints in maximum flow problems, allowing for capacity violations within tolerance ranges. They developed efficient algorithms for integer flows and proved a theorem equivalent to Ford-Fulkerson's for fuzzy networks. Hernandes et al. (2007) proposed an algorithm based on Ford-Fulkerson for fuzzy maximum flow problems, using an incremental graph approach that doesn't require users to specify a desired flow. This is particularly useful for large-scale networks. Teodorovic and Selmic (2013) applied fuzzy set theory to locate flow-capturing facilities in transportation networks, treating estimated trip numbers as fuzzy numbers. Their model maximizes intercepted client flow using fuzzy mathematical programming. These studies demonstrate the versatility of fuzzy set theory in addressing uncertainties in network capacities and flow estimations across various applications, including telecommunications, transportation, and manufacturing.

II. RELATED WORK

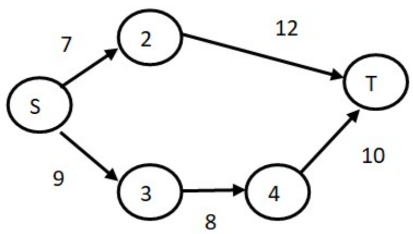

Fig. 2.1 The initial capacity network

Iteration 1:
Step 1: Select the path from source to sink node
S—3—4—T
Step 2: capacities along the arcs are
S—3 = 9
3—4 = 8
4—T = 10
Step 3: Find C_1 = min{(S—3), (3—4), (4—T)}
$C_1 = 9$
Step 4: Update the capacities of {(S—3), (3—4), (4—T)} (by subtracting the $C_1 = 9$)
Step 5: Update the capacities of {(S—3), (3—4), (4—T)} (by adding the $C_1 = 9$)
Step 6: After updating the capacities, the diagram will be

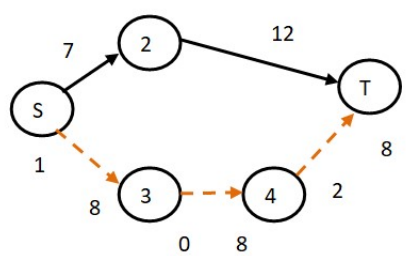

Fig. 2.2 The flow graph after first iteration

Iteration 2:
Step 1: Select the path from source to sink node
S—2—T
Step 2: capacities along the arcs are
S—2 = 7
2—T = 12
Step 3: Find C_2 = min{(S—2), (2—T)}
$C_2 = 7$
Step 4: Update the capacities of {(S—2), (2—T)} (by subtracting the $C_2 = 7$)
Step 5: Update the capacities of {(S—2), (2—T)} (by adding the $C_2 = 7$)
Step 6: After updating the capacities, the diagram will be

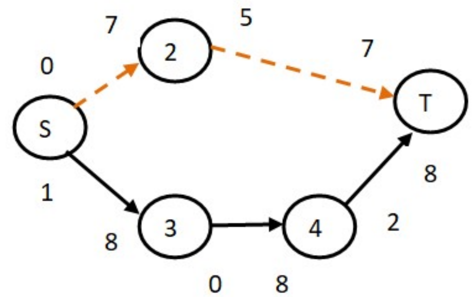

Fig. 2.3 The flow graph after second iteration

Then, the final flow diagram will be

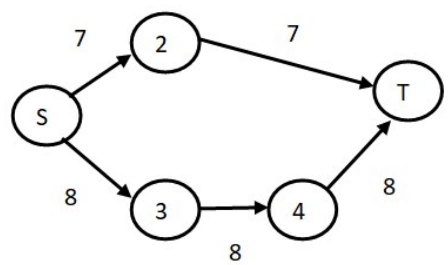

Fig. 2.4 The Flow network

So, the maximum flow of the diagram will be 15.

III. METHODOLOGY

Step 1: Initialize the fuzzy sets A and B over the universal set X = $\{x_1, x_2, x_3, x_4, x_5\}$.

Step 2: Calculate the cardinality of set A (|A|) as the sum of the membership values and the cardinality of set B (|B|) as the sum of the membership values.

Step 3: Calculate the union of sets A and B (AUB) by taking the maximum membership value for each element:
|AUB| = max $\{(Ax_1, Bx_1) + (Ax_2, Bx_2) + (Ax_3, Bx_3) + (Ax_4, Bx_4) + (Ax_5, Bx_5)\}$ and the sum of the cardinalities of sets A and B: |A| + |B|

Step 4: Compare the union of sets A and B (|AUB|) with the sum of their cardinalities (|A| + |B|) to check for inequality:
AUB ≠ |A| + |B|" if |AUB| is not equal to |A| + |B|

Step 5: Calculate the Cartesian product of sets A and B (A X B) by taking the product of each pair of minimum membership values:
|A X B| = min$\{(Ax_1, Bx_1) + (Ax_2, Bx_2) + (Ax_3, Bx_3) + (Ax_4, Bx_4) + (Ax_5, Bx_5)\}$ and the product of the cardinalities of sets A and B: |A| |B|

Step 6: Compare the Cartesian product of sets A and B (|A X B|) with the product of their cardinalities (|A| |B|) to check for inequality: "A X B| ≠ |A| |B|" if |A X B| is not equal to |A| |B|

Step 7: Identify the multiple set M defined over $X=\{x_1, x_2, x_3, x_4, x_5\}$ with a membership matrix of order 5×5.

Step 8: Create the membership matrices $M(x_1)$ and $M(x_2)$. The membership data is to be filled according to specified rules or data inputs. Sum all contributions in the membership matrix.

Step 9: Calculate the average membership per element:

$$\text{Average Membership} = \frac{\|M\|}{\text{Total Elements}}$$

Step 10:

Let |A| representing overall capacity.
Let |B| representing flow through the system and determine the difference between capacity and flow: Capacity – Flow.

Step 11: Calculate $n = \|M\| - |B|$ and the maximum flow value is $2^n - 1$

Consider two fuzzy set A and B defined over the universal set. $X = \{x_1, x_2, x_3, x_4, x_5\}$ given by $A = \{(x_1,7), (x_2, 9), (x_3,12), (x_4,8), (x_5,10)\}$ and $B = \{(x_1,7), (x_2, 8), (x_3,7), (x_4,8), (x_5,8)\}$ Find |A|, |B|, |AUB|, |A| + |B|, |AUB| ≠|A| + |B| and |A X B| ≠ |A| |B|.

|A| = 46, |B| = 38

$|AUB| = \max\{(Ax_1, Bx_1) + (Ax_2, Bx_2) + (Ax_3, Bx_3) + (Ax_4, Bx_4) + (Ax_5, Bx_5)\}$

$= \max\{(7,7) + (9,8) + (12,7) + (8,8) + (10,8)\}$

$= 46$

|A| + |B| = 84

|AUB| ≠|A| + |B|

$|A \times B| = \min\{(Ax_1, Bx_1) + (Ax_2, Bx_2) + (Ax_3, Bx_3) + (Ax_4, Bx_4) + (Ax_5, Bx_5)\}$

$= \min\{(7,7) + (9,8) + (12,7) + (8,8) + (10,8)\}$

= 25,088

|A| |B| = 1748

∴|A X B| ≠ |A| |B|

Consider a multiple set M of order (5 x 5) defines over $X = \{x_1, x_2, x_3, x_4, x_5\}$ whose membership matrix is given below.

$$M(x_1) = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 7 & 9 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 12 \\ 3 & 0 & 0 & 0 & 8 & 0 \\ 4 & 0 & 0 & 0 & 0 & 10 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Where $M(x_1)$ is Capacity Matrix

$$M(x_2) = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 4 & 3 & 0 \\ 2 & 0 & 0 & 3 & 4 \\ 3 & 0 & 0 & 0 & 5 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

Where $M(x_2)$ is the Flow Matrix.

| M | = 84

$\|M\| = 42$

IV MAXIMUM FLOW EXAMPLE

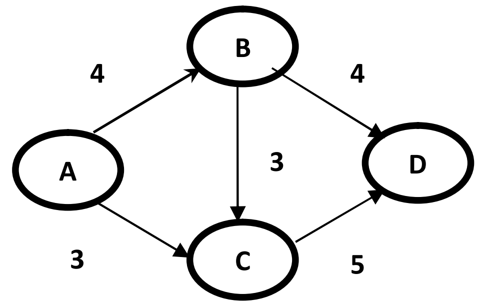

Fig. 4.1. Capacity Diagram

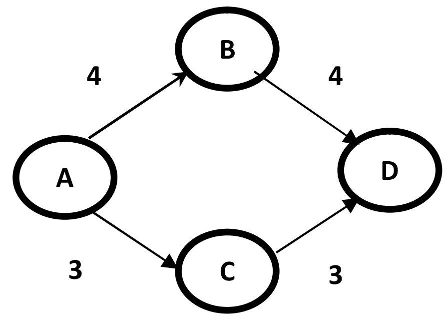

Fig. 4.2. Flow Diagram

Consider two fuzzy sets A and B defined over the universal set. $X = \{x_1, x_2, x_3, x_4, x_5\}$ given by $A = \{(x_1,4), (x_2, 3), (x_3,3), (x_4,4), (x_5,5)\}$ and $B = \{(x_1,4), (x_2, 3), (x_3,0), (x_4,4), (x_5,3)\}$

Find |A|, |B|, |AUB|, |A| + |B|, |AUB| ≠|A| + |B| and |A X B| = |A| |B|.

|A| = 19

|B| = 14

$|AUB| = \max\{(Ax_1, Bx_1) + (Ax_2, Bx_2) + (Ax_3, Bx_3) + (Ax_4, Bx_4) + (Ax_5, Bx_5)\}$

$= \max\{(4,4) + (3,3) + (3,0) + (4,4) + (5,3)\}$

$= 19$

|A| + |B| = 33

|AUB| ≠|A| + |B|

$|A \times B| = \min\{(Ax_1, Bx_1) + (Ax_2, Bx_2) + (Ax_3, Bx_3) + (Ax_4, Bx_4) + (Ax_5, Bx_5)\}$

$= \min\{(4,4) + (3,3) + (3,0) + (4,4) + (5,3)\}$

= 0

|A| |B| = 266

∴|A X B| ≠ |A| |B|

Consider a multiple set M of order (5 x 5) defines over $X = \{x_1, x_2, x_3, x_4, x_5\}$ whose membership matrix is given below.

$$M(x_1) = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 7 & 8 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 7 \\ 3 & 0 & 0 & 0 & 8 & 0 \\ 4 & 0 & 0 & 0 & 0 & 8 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$M(x_2) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 4 & 3 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$|M| = 33$

$\|M\| = 16.5$

$n = \|M\| - |B|$

$\approx$ **3**

$= 2^n - 1$

$= 2^3 - 1$

Therefore, the Maximum Flow value = 7.

V. RESULT AND DISCUSSION

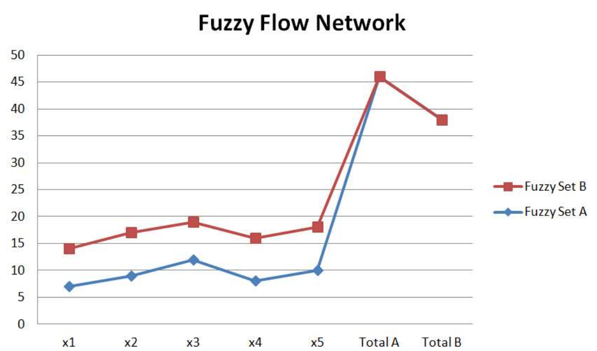

Fig. 5.1. The relation between Capacity and Flow Diagram.

Figure 5.1 depicts the fuzzy sets A and B have cardinalities of 46 and 38, respectively. However, their union has a cardinality of 46, which is less than the sum of their individual cardinalities (84). This discrepancy highlights that combining fuzzy sets doesn't always equal the sum of their parts. Furthermore, the Cartesian product of A and B has a cardinality of 25,088, significantly differing from the product of their individual cardinalities (1,748). This illustrates that the cardinality of the Cartesian product doesn't necessarily match the product of individual cardinalities.

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