On n-Dimensional ARA Integral Transform

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Abstract: This paper describes the construction of n dimensional ARA integral transform. We use the technique of extending the one dimensional integral transform to double integral transform. This n dimensional ARA integral transform can be used to solve partial differential equations of any order including heat equation, wave equation, telegraph equation and others, which are mainly used in physical sciences.

Key words: ARA transforms, Double ARA transform, Differential equation, Convolution.

AMS Subject Classification: 42A38, 42B10, 26A33

Introduction: The use of double transformation is considered extremely for solving ordinary and partial differential equations. There are lots of double transforms such as double Natural transform [7], double Elzaki transform, double Laplace transform and so on. Saadeh and others recently introduced the ARA transform [2]. Giang Bui Thi [8] gave operational properties of two Integral Transforms of Fourier Type and their convolution; Mahajan and Choudhary extended this Fourier Type Transforms and Their Convolutions on $\mathbb{R}^n$ [9]. Khan, Elzaki and Saadeh gave various integral transforms and their properties with suitable kernels in[3], [4], [5], respectively. Raania Saadeh gave Double ARA-Sumudu Transform and its Applications & Application of Double ARA Integral Transform in [6] and [1] respectively.

This paper describes and proves the theorem and properties of n Dimensional ARA Integral Transform. Many of the researchers have studied the solution of partial differential equation and one of the most important techniques used to develop is transform method. Here we give some new relations with n dimensional ARA transform and partial derivative.

The structure of this paper includes following sections
Section 1: Definition and theorem of one dimensional and double ARA integral transform.
Section 2: Definition of n dimensional ARA integral transform
Section 3: Properties of n dimensional ARA integral transform
Section 4: Conclusion

Section 1:

Definition: The ARA integral transform of nth order of a continuous function $p(x)$ on $(0, \infty)$ is defined by [2] and is given by $A_n[p(t)](s) = s \int_0^\infty t^{n-1} e^{-st} p(t)dt$ $\cdots$ [1]

If $n = 1$ then $A_1[p(t)](s) = s \int_0^\infty e^{-st} p(t)dt$ $\cdots$ [2]

The inversion theorem for one dimensional ARA transform is given in lemma 1 and theorem 1 of [2]

The two dimensional or double ARA transform of two variables x_1, x_2 of a continuous function $p(x_1, x_2)$ is defined by [2] as,

$$A_{x_1,x_2}\{p(x_1,x_2)\} = p(s_1,s_2) = s_1 s_2 \int_0^\infty \int_0^\infty e^{-(s_1x_1+s_2x_2)} p(x_1,x_2)\ dx_1 dx_2$$

Section 2:

Definition: The n dimensional ARA integral transform of continuous function of n variables $p(x_1, x_2, \dots, x_n)$ is defined as

$$A_{x_1}A_{x_2} \dots A_{x_n}\{p(x_1,x_2,\dots,x_n)\} = p(s_1,s_2,\dots,s_n)$$
$$= s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1+s_2x_2+\dots+s_nx_n)} p(x_1,x_2,\dots,x_n)\ dx_1 dx_2 \dots dx_n$$

Clearly the n dimensional ARA integral transform is linear integral transformation

$$A_{x_1}A_{x_2} \dots A_{x_n}\{c_1\, p_1(x_1,x_2,\dots,x_n) + c_2\, p_2(x_1,x_2,\dots,x_n)\}$$
$$= s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1+s_2x_2+\dots+s_nx_n)} [c_1\, p_1(x_1,x_2,\dots,x_n) + c_2\, p_2(x_1,x_2,\dots,x_n)]\ dx_1 dx_2 \dots dx_n$$

$$= c_1 s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1+s_2x_2+\dots+s_nx_n)}\ p_1(x_1,x_2,\dots,x_n) dx_1 dx_2 \dots dx_n$$
$$+ c_2 s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1+s_2x_2+\dots+s_nx_n)}\ p_2(x_1,x_2,\dots,x_n)\ dx_1 dx_2 \dots dx_n$$

$$= c_1 A_{x_1}A_{x_2} \dots A_{x_n}[p_1(x_1,x_2,\dots,x_n)] + c_2 A_{x_1,x_2,\dots,x_n}[p_2(x_1,x_2,\dots,x_n)]$$

where c_1 and c_2 are constants and $A_{x_1}A_{x_2} \dots A_{x_n}[p_1(x_1,x_2,\dots,x_n)]$ and $A_{x_1,x_2,\dots,x_n}[p_2(x_1,x_2,\dots,x_n)]$ exists.

Section 3:

Property 1: Let $p(x_1,x_2,\dots,x_n) = q_1(x_1)q_2(x_2)\dots q_n(x_n), x_i > 0, i = 1,2,\dots,n$. Then

$$A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1,x_2,\dots,x_n)] = A_{x_1}[q_1(x_1)]A_{x_2}[q_2(x_2)]\dots A_{x_n}[q_n(x_n)]$$

Proof: $A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1,x_2,\dots,x_n)] = A_{x_1}A_{x_2} \dots A_{x_n}[q_1(x_1)q_2(x_2)\dots q_n(x_n)]$

$$= s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}\, q_1(x_1)q_2(x_2)\dots q_n(x_n)dx_1dx_2\dots dx_n$$

$$= s_1 \int_0^\infty e^{-s_1x_1}\, q_1(x_1)\, dx_1 \cdot s_2 \int_0^\infty e^{-s_2x_2}\, q_2(x_2)\, dx_2 \cdot \dots \cdot s_n \int_0^\infty e^{-s_nx_n}\, q_n(x_n)\, dx_n$$

$$= A_{x_1}[q_1(x_1)]A_{x_2}[q_2(x_2)]\dots A_{x_n}[q_n(x_n)]$$

Property 2: Let $p(x_1, x_2, \dots, x_n) = 1, x_i > 0, i = 1, 2, \dots, n$. Then, $A_{x_1}A_{x_2} \dots A_{x_n}[1] = 1$

Proof: $A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n)] = s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}\, dx_1dx_2\dots dx_n$

$$= s_1 \int_0^\infty e^{-s_1x_1}\, dx_1 \cdot s_2 \int_0^\infty e^{-s_2x_2}\, dx_2 \cdot \dots \cdot s_n \int_0^\infty e^{-s_nx_n}\, dx_n$$

$$= A_{x_1}[1]A_{x_2}[1]\dots A_{x_n}[1] = 1$$

Where $Re(x_1) > 0, Re(x_2) > 0 \dots Re(x_n) > 0$

Property 3: Let $p(x_1, x_2, \dots, x_n) = {x_1}^{\alpha_1}{x_2}^{\alpha_2} \dots {x_n}^{\alpha_n}, x_i > 0, i = 1, 2, \dots, n$ and $\alpha_1, \alpha_2, \dots, \alpha_n$ are constants. Then, $A_{x_1}A_{x_2} \dots A_{x_n}[{x_1}^{\alpha_1}{x_2}^{\alpha_2} \dots {x_n}^{\alpha_n}] = A_{x_1}[{x_1}^{\alpha_1}]A_{x_2}[{x_2}^{\alpha_2}] \dots A_{x_n}[{x_n}^{\alpha_n}]$

Proof: $A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n)] = A_{x_1}A_{x_2} \dots A_{x_n}[{x_1}^{\alpha_1}{x_2}^{\alpha_2} \dots {x_n}^{\alpha_n}]$

$$= s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}\, [{x_1}^{\alpha_1}{x_2}^{\alpha_2} \dots {x_n}^{\alpha_n}]\, dx_1dx_2\dots dx_n$$

$$= s_1 \int_0^\infty e^{-s_1x_1}\, [{x_1}^{\alpha_1}]dx_1 \cdot s_2 \int_0^\infty e^{-s_2x_2}\, [{x_2}^{\alpha_2}]dx_2 \cdot \dots \cdot s_n \int_0^\infty e^{-s_nx_n}\, [{x_n}^{\alpha_n}]\, dx_n$$

$$= A_{x_1}[{x_1}^{\alpha_1}]A_{x_2}[{x_2}^{\alpha_2}] \dots A_{x_n}[{x_n}^{\alpha_n}]$$

From the properties of ARA transform, we obtain,

$$A_{x_1}A_{x_2} \dots A_{x_n}[{x_1}^{\alpha_1}{x_2}^{\alpha_2} \dots {x_n}^{\alpha_n}] = A_{x_1}[{x_1}^{\alpha_1}]A_{x_2}[{x_2}^{\alpha_2}] \dots A_{x_n}[{x_n}^{\alpha_n}] = \frac{\Gamma(\alpha_1+1)(\alpha_2+1)\dots(\alpha_n+1)}{{s_1}^{\alpha_1}{s_2}^{\alpha_2} \dots {s_n}^{\alpha_n}},$$

$\mathrm{Re}(\alpha_1) > -1, \mathrm{Re}(\alpha_2) > -1, \dots, \mathrm{Re}(\alpha_n) > -1$

Property 4: Let $p(x_1, x_2, \dots, x_n) = e^{\alpha_1x_1+\alpha_2x_2+\cdots+\alpha_nx_n}, x_i > 0, i = 1, 2, \dots, n$ and $\alpha_1, \alpha_2, \dots, \alpha_n$ are constants. Then, $A_{x_1}A_{x_2} \dots A_{x_n}[e^{\alpha_1x_1+\alpha_2x_2+\cdots+\alpha_nx_n}] = A_{x_1}[e^{\alpha_1x_1}]A_{x_2}[e^{\alpha_1x_1}] \dots A_{x_n}[e^{\alpha_1x_1}]$

$$A_{x_1}A_{x_2} \dots A_{x_n}[e^{\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_n x_n}]$$
$$= s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}\,[e^{\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_n x_n}]\,dx_1dx_2 \dots dx_n$$

$$= s_1 \int_0^\infty e^{-s_1x_1}\,[e^{\alpha_1x_1}]dx_1 \cdot s_2 \int_0^\infty e^{-s_2x_2}\,[e^{\alpha_2x_2}]dx_2 \cdot \dots \cdot s_n \int_0^\infty e^{-s_nx_n}\,[e^{\alpha_nx_n}]\,dx_n$$

$$= A_{x_1}[e^{\alpha_1x_1}]A_{x_2}[e^{\alpha_1x_1}] \dots A_{x_n}[e^{\alpha_1x_1}]$$

From the properties of ARA transform, we obtain,

$$A_{x_1}A_{x_2} \dots A_{x_n}[e^{\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_n x_n}] = \frac{s_1s_2 \dots s_n}{(s_1-\alpha_1)(s_2-\alpha_2)\dots(s_n-\alpha_n)}$$

Property 5 (Shifting Property): Let $p(x_1, x_2, \dots, x_n)$ be a continuous function and $A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n)] = P(s_1, s_2, \dots, s_n)$ then

$$A_{x_1}A_{x_2} \dots A_{x_n}[e^{\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_n x_n}p(x_1, x_2, \dots, x_n)]$$
$$= \frac{s_1s_2 \dots s_n}{(s_1-\alpha_1)(s_2-\alpha_2)\dots(s_n-\alpha_n)}\,P\big((s_1-\alpha_1), (s_2-\alpha_2), \dots, (s_n-\alpha_n)\big)$$

Proof: From the definition of n dimensional ARA integral transform,

$$A_{x_1}A_{x_2} \dots A_{x_n}[e^{\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_n x_n}p(x_1, x_2, \dots, x_n)]$$
$$= s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}e^{\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_n x_n}[p(x_1, x_2, \dots, x_n)]\;dx_1dx_2 \dots dx_n$$

$$= s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1-\alpha_1)x_1-(s_2-\alpha_2)x_2-\cdots-(s_n-\alpha_2)x_n}[p(x_1, x_2, \dots, x_n)]\;dx_1dx_2 \dots dx_n$$

$$= s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1-\alpha_1)x_1}e^{-(s_2-\alpha_2)x_2} \dots e^{-(s_n-\alpha_n)x_n}[p(x_1, x_2, \dots, x_n)]\;dx_1dx_2 \dots dx_n$$

$$= \frac{s_1s_2 \dots s_n}{(s_1-\alpha_1)(s_2-\alpha_2)\dots(s_n-\alpha_n)}P\big((s_1-\alpha_1), (s_2-\alpha_2), \dots, (s_n-\alpha_n)\big)$$

Property 6 (Periodic Function): Let $A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n)]$ exist, where $p(x_1, x_2, \dots, x_n)$ describes a periodic function of periods $\alpha_1, \alpha_2, \dots, \alpha_n$ such that

$p(x_1+\alpha_1, x_2+\alpha_2, \dots, x_n+\alpha_n) = p(x_1, x_2, \dots, x_n), \forall x_i, i = 1, 2, \dots, n$ then,

$$A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1,x_2,\dots,x_n)]$$
$$= \frac{1}{(1-e^{-(s_1\alpha_1+s_2\alpha_2+\cdots+s_n\alpha_n)})}\left(s_1s_2\dots s_n\int_0^{\alpha_1}\int_0^{\alpha_2}\dots\int_0^{\alpha_n} e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}[p(x_1,x_2,\dots,x_n)]\ dx_1dx_2\dots dx_n\right)$$

Proof: By definition of n dimensional ARA integral transform,

$$A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1,x_2,\dots,x_n)] = s_1s_2\dots s_n\int_0^{\infty}\int_0^{\infty}\dots\int_0^{\infty} e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}p(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n$$
$$= s_1s_2\dots s_n\int_0^{\alpha_1}\int_0^{\alpha_2}\dots\int_0^{\alpha_n} e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}p(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n$$
$$+ s_1s_2\dots s_n\int_{\alpha_1}^{\infty}\int_{\alpha_2}^{\infty}\dots\int_{\alpha_n}^{\infty} e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}p(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n$$

Put $x_1=\alpha_1+r_1, x_2=\alpha_2+r_2,\dots,x_n=\alpha_n+r_n$

$$= s_1s_2\dots s_n\int_0^{\alpha_1}\int_0^{\alpha_2}\dots\int_0^{\alpha_n} e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}p(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n$$
$$+ s_1s_2\dots s_n\int_0^{\infty}\int_0^{\infty}\dots\int_0^{\infty} e^{-(s_1(\alpha_1+r_1)+s_2(\alpha_2+r_2)+\cdots+s_n(\alpha_n+r_n))}p\big((\alpha_1+r_1),(\alpha_2+r_2),\dots,(\alpha_n+r_n)\big)\ dr_1dr_2\dots dr_n$$

Using the periodicity of the function $p(x_1,x_2,\dots,x_n)$ above equation can be written as

$$P(s_1,s_2,\dots,s_n) = s_1s_2\dots s_n\int_0^{\alpha_1}\int_0^{\alpha_2}\dots\int_0^{\alpha_n} e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}p(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n$$
$$+ e^{-(s_1\alpha_1+s_2\alpha_2+\cdots+s_n\alpha_n)}s_1s_2\dots s_n\int_0^{\infty}\int_0^{\infty}\dots\int_0^{\infty} e^{-(s_1r_1+s_2r_2+\cdots+s_nr_n)}p(r_1,r_2,\dots,r_n)\ dr_1dr_2\dots dr_n$$

$$P(s_1,s_2,\dots,s_n) = s_1s_2\dots s_n\int_0^{\alpha_1}\int_0^{\alpha_2}\dots\int_0^{\alpha_n} e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}p(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n$$
$$+ e^{-(s_1\alpha_1+s_2\alpha_2+\cdots+s_n\alpha_n)}P(s_1,s_2,\dots,s_n)$$

$$P(s_1,s_2,\dots,s_n)\big[1-e^{-(s_1\alpha_1+s_2\alpha_2+\cdots+s_n\alpha_n)}\big] =$$
$$s_1s_2\dots s_n\int_0^{\alpha_1}\int_0^{\alpha_2}\dots\int_0^{\alpha_n} e^{-(s_1x_1+s_2x_2+\cdots+s_nx_n)}p(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n$$

$$P(s_1, s_2, \dots, s_n) = \frac{1}{(1 - e^{-(s_1\alpha_1 + s_2\alpha_2 + \dots + s_n\alpha_n)})} \left(s_1 s_2 \dots s_n \int_0^{\alpha_1} \int_0^{\alpha_2} \dots \int_0^{\alpha_n} e^{-(s_1x_1 + s_2x_2 + \dots + s_nx_n)} [p(x_1, x_2, \dots, x_n)]\ dx_1 dx_2 \dots dx_n \right)$$

Property 7 (Heaviside Function): Let $A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n)]$ exist and $A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n)] = P(s_1, s_2, \dots, s_n)$ then $A_{x_1}A_{x_2} \dots A_{x_n}[p[(x_1 - \delta_1), (x_2 - \delta_2), \dots, (x_n - \delta_n)]\mathrm{H}((x_1 - \delta_1), (x_2 - \delta_2), \dots, (x_n - \delta_n))] = e^{-s_1\delta_1 - s_2\delta_2 - \dots - s_n\delta_n} P(s_1, s_2, \dots, s_n)$
where $\mathrm{H}((x_1 - \delta_1), (x_2 - \delta_2), \dots, (x_n - \delta_n))$ is Heaviside unit step function defined by

$$\mathrm{H}((x_1 - \delta_1), (x_2 - \delta_2), \dots, (x_n - \delta_n)) = \begin{cases} 1, & \text{for } x_i > \delta_i, i = 1, 2, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

Proof: By using the definition, we have

$$A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n)] = s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1 + s_2x_2 + \dots + s_nx_n)}\, p[(x_1 - \delta_1), (x_2 - \delta_2), \dots, (x_n - \delta_n)]\mathrm{H}((x_1 - \delta_1), (x_2 - \delta_2), \dots, (x_n - \delta_n))\, dx_1 dx_2 \dots dx_n$$

$$= s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(s_1x_1 + s_2x_2 + \dots + s_nx_n)}\, p[(x_1 - \delta_1), (x_2 - \delta_2), \dots, (x_n - \delta_n)]\, dx_1 dx_2 \dots dx_n$$

Put $x_i - \delta_i = r_i, i = 1, 2, \dots, n$ in above equation

$$A_{x_1}A_{x_2} \dots A_{x_n}[p[(x_1 - \delta_1), (x_2 - \delta_2), \dots, (x_n - \delta_n)]\mathrm{H}((x_1 - \delta_1), (x_2 - \delta_2), \dots, (x_n - \delta_n))]$$

$$= s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-s_1(\delta_1 + r_1) - s_2(\delta_2 + r_2) - \dots - s_n(\delta_n + r_n)}\, p(r_1, r_2, \dots, r_n)\, dr_1 dr_2 \dots dr_n$$

$$= e^{-s_1\delta_1 - s_2\delta_2 - \dots - s_n\delta_n} \left(s_1 s_2 \dots s_n \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-s_1r_1 - s_2r_2 - \dots - s_nr_n}\, p(r_1, r_2, \dots, r_n)\, dr_1 dr_2 \dots dr_n \right)$$

$$= e^{-s_1\delta_1 - s_2\delta_2 - \dots - s_n\delta_n} P(s_1, s_2, \dots, s_n)$$

Property 8 (Convolution Theorem): Let $A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n)]$ and $A_{x_1}A_{x_2} \dots A_{x_n}[q(x_1, x_2, \dots, x_n)]$ exist and $A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n)] = P(s_1, s_2, \dots, s_n)$, $A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n)] = Q(s_1, s_2, \dots, s_n)$ then,
$A_{x_1}A_{x_2} \dots A_{x_n}[p(x_1, x_2, \dots, x_n) ** q(x_1, x_2, \dots, x_n)] = \frac{1}{s_1 s_2 \dots s_n} P(s_1, s_2, \dots, s_n) Q(s_1, s_2, \dots, s_n)$

Where
$p(x_1, x_2, \dots, x_n) ** q(x_1, x_2, \dots, x_n) =$

$\int_0^{x_1} \int_0^{x_2} \dots \int_0^{x_n} p(x_1 - r_1, x_2 - r_2, \dots, x_n - r_n)\, q(r_1, r_2, \dots, r_n)\, dr_1 dr_2 \dots dr_n$ and $**$ denotes the convolution with respect to $x_1, x_2, \dots, x_n$.

The proof of this theorem can be verified easily using the similar proof of double ARA integral transform.

Section 4:

Conclusion: In this paper we have given the definition and the properties of n dimensional ARA integral transform such as linearity property, shifting property, periodic function, Heaviside function. Also we have given the convolution theorem for n dimensional ARA integral transform. For future study we will apply this n dimensional ARA integral transform to solve PDEs and higher order ordinary differential equations.

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