Computational Method to Study Mie Scattering by Magnetic Scatterers in Magnetic Colloids

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Abstract.

Mie scattering is a technique used to investigate the scattering of light by particles that are larger than the wavelength of light. The Mie algorithm is a mathematical tool used to calculate different scattering parameters. To obtain the scattering and extinction parameters for Mie scattering, it is necessary to have the Mie coefficients a_n and b_n. This research examines the phenomenon of light scattering by magnetic particles in a magnetic fluid. We present a novel algorithm for determining the Mie coefficients a_n and b_n, which describe the scattering of light by a magnetic sphere submerged in a magnetic fluid. This work facilitates the determination of scattering parameters, such as the scattering cross section (σ_{sca}), extinction cross section (σ_{ext}), and absorption cross section (σ_{abs}), for magnetic spheres that are enveloped by a magnetic medium.

Keywords: Mie scattering, Mie coefficient, Magnetic fluid, Scattering and extinction cross section

Introduction

Light scattering is a highly promising area of research in the science of optical physics. Scattering refers to the phenomenon when particles in a material absorb light and then re-emit it in various directions. The scattering intensity is determined by the particle size parameter (x) and the wavelength of light (λ). Scattering that occurs when the size of a particle is smaller than the wavelength of light ($x < \lambda$) is referred to as Rayleigh's scattering. On the other hand, when the size of the particle is larger than the wavelength of light ($x > \lambda$), it is known as Lorentz-Mie scattering.[1][2] The intensity of small particle scattering is inversely proportional to the fourth power of λ. Calculating the intensity for scattering of big particles is a complicated task. Lorentz and Mie devised a theory of electromagnetic scattering that applies to homogeneous spheres of any size. [3][4] The Lorentz-Mie solution is valid for a spherical particle that is surrounded by a homogeneous and isotropic medium which has complex refractive index that varies with frequency. The scattering, extinction, absorption, and total cross sections are calculated based on the Mie scattering coefficients a_n and b_n.[1]

Ferrofluid (FF), also known as magnetic fluid, is a type of colloidal suspension where magnetic nanoparticles are suspended in a liquid carrier that is compatible with them. In order to prevent the clustering of particles, a layer of surfactants is applied to them. [5] FF possesses distinctive characteristics of both liquidity and magnetism. FF exhibits intriguing optical effects when subjected to an external magnetic field, such as forward and backward scattering, as well as optical birefringence.[6], [7] These investigations demonstrate that FF is an excellent choice for investigating applications such as biosensors, photonic band gap materials, smart dampers and sensors, optical grating, smart switches, smart filters, and more.[8]–[13]

Lorentz-Mie Scattering of Ferrofluid

Typically, light scattering investigations focus on non-magnetic scatterers that are surrounded by a non-magnetic medium. [14]–[16] The phenomenon of scattering caused by a magnetic particle that is surrounded by non-magnetic material has been investigated by multiple researchers [17]–[21]. The investigation of light scattering in magnetic particles surrounded by magnetically active mediums remains an untouched field in magneto-optics. In their study, M. Kerker et al. documented the specific characteristics of light scattering by magnetic spheres. [22] The analysis of light scattering by a spherical particle in a homogeneous and isotropic medium, when it is irradiated by a linearly polarized plane wave, can be conducted using the Lorentz-Mie theory.[3] The Mie theory can be approximated up to dipolar terms in the multipolar expansion, known as the dipolar approximation[23], when dealing with small particle limits. The investigation on light scattering for FF in the presence of an external magnetic field demonstrates that the normalized transmitted intensity falls as the magnetic field increases, reaching a minimum value at a specific field strength, and then increases again.[24] This response suggests the presence of structural anisotropy in FF when exposed to a magnetic field.

Computation of Scattering Parameters

This research presents a computational model that investigates the scattering properties of a nanomagnetic fluid containing suspended micron-sized magnetic particles. The Lorentz-Mie theory can be used to determine the scattering amplitudes (S_1 and S_2), efficiencies (Q_{sca}, Q_{ext}, and Q_{abs}), and cross sections (σ_{sca}, σ_{ext}, and σ_{abs}) for the scattering of light by magnetic spheres in a magnetizable medium. The algorithm for calculating the such parameters is given below:

(i) Find field dependent refractive index (η) by using $\eta = \eta_\infty + L(\xi)\,\eta_0$; where, η_∞ is refractive index at saturation field and η_0 is refractive index at zero field, $L(\xi)$ is Langevin function given by $L(\xi) = \coth(\xi) - \frac{1}{\xi}$ and $\xi = \frac{mH}{kT}$, m is magnetic moment, H is applied field, k is Boltzmann constant and T is temperature.[25]

(ii) Using field dependent refractive index, find Mie-coefficients a_n and b_n by using following formulae[1]

$$a_n = \frac{\mu\psi'_n(y)\psi_n(x) - \eta\psi_n(y)\psi'_n(x)}{\mu\psi'_n(y)\zeta_n(x) - \eta\psi_n(y)\zeta'_n(x)},$$

$$b_n = \frac{\eta\psi'_n(y)\psi_n(x) - \mu\psi_n(y)\psi'_n(x)}{\eta\psi'_n(y)\zeta_n(x) - \mu\psi_n(y)\zeta'_n(x)}$$

Where, ψ_n and ζ_n are Riccati-Bessel and Hankel functions respectively, $x = k\, r,$ where r is radius of the particle, $k = 2\pi / \lambda$ and $y = \eta\, x,$ η is field dependent refractive index.

(iii) With the help of a_n and b_n, we can calculate Mie angular functions,

$$\pi_n(\cos\theta) = \frac{1}{\sin\theta} P_n'(\cos\theta),$$

$$\tau_n(\cos\theta) = \frac{d}{d\theta}[P_n'(\cos\theta)]$$

Where, θ is the angle between the forward and scattering directions and P_n' are associated Legendre functions.

(iv) With the help of a_n, b_n, π_n and τ_n one can calculate Mie scattering amplitudes,

$$S_1(n_s, x, \theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \pi_n(\cos\theta) + b_n \tau_n(\cos\theta)],$$

$$S_2(n_s, x, \theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \tau_n(\cos\theta) + b_n \pi_n(\cos\theta)]$$

(v) With the help of Mie co-efficients a_n and b_n one can find Extinction efficiency (Q_{ext}), Scattering efficiency (Q_{sca}) and Absorption efficiency (Q_{abs}) by using following formulae,

$$Q_{ext} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \mathrm{Re}(a_n + b_n),$$

$$Q_{sca} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2)$$

$$Q_{abs} = Q_{ext} - Q_{sca}$$

(vi) One can calculate scattering, extinction, and absorption cross sections by

$$\sigma_{ext} = \frac{\pi r^2}{2} Q_{ext}$$
$$\sigma_{sca} = \frac{\pi r^2}{2} Q_{sca}$$
$$\sigma_{abs} = \frac{\pi r^2}{2} Q_{abs}$$

Here, $n = 1, 2, \ldots N$ and $N \approx x + 4\, x^{1/3} + 2$[26].

Conclusion

The investigation of light scattering in magnetic particles surrounded by magnetically active medium is a developing field in magneto-optics. The scattering properties of such a system can be adjusted by the application of an external magnetic field. This paper presents an algorithm that is developed to calculate scattering amplitudes, scattering, extinction, and absorption efficiencies as well as cross sections. This study urges researchers to create such a system in a controlled laboratory setting. A magnetically adjustable colloidal system has potential applications in the advancement of photonic band gap materials, optical fiber communication, smart materials, and other fields.

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