Face and Total Face Product Cordial Labeling in Some Graph Families

K. K. Raval[1], A. B. Joshi[1,*], N. B. Patel[2]

[1] Department of Mathematics, Gujarat University, Ahmedabad, Gujarat, INDIA.
[1,*]Science and Humanities Department, Silver Oak College of Engineering and Technology,
Ahmedabad, Gujarat, INDIA.
[2]Science and Humanities Department, Sal Institute of Technology and Engineering Research,
Ahmedabad, Gujarat, INDIA.

Abstract

For a planar graph G, the vertex labeling function is defined as $g: V(G) \rightarrow \{0, 1\}$ and $g(v)$ is called the label of the vertex v of G under g, induced edge labeling function $g^*: E(G) \rightarrow \{0, 1\}$ is given as if $e = uv$ then $g^*(uv) = g(u)g(v)$ and induced face labeling function $g^{**}: F(G) \rightarrow \{0, 1\}$ is given as if $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_m$ are the vertices and edges of face f then $g^{**}(f) = g(v_1)g(v_2) \ldots g(v_n)g^*(e_1)g^*(e_2) \ldots g^*(e_m)$. Let us denote $v_g(i)$ is the number of vertices of G having label i under g, $e_g(i)$ is the number of edges of G having label i under g^* and $F_g(i)$ is the number of interior faces of G having label i under g^{**}, for i $\in \{0, 1\}$. g is called the face product cordial labeling of graph G if $|v_g(0) - v_g(1)| \leq 1$, $|e_g(0) - e_g(1)| \leq 1$ akd $|F_g(0) - F_g(1)| \leq 1$. A graph G is a face product cordial graph if it admits face product cordial labeling. Let $g(0)$ and $g(1)$ be the sum of the number of vertices, edges and interior faces having labels 0 and labels 1 respectively. g is called total face product cordial labeling of graph G if $|g(0) - g(1)| \leq 1$. A graph G is called total face product cordial graph if it admits total face product cordial labeling. Face product cordial labeling is investigated for book graph when k is odd and $m \leq 2$. Face product cordial and total face product cordial labeling of gear graph is explored when $k \geq 3$. Also, the graph obtained from a gear graph by duplicating each vertex of degree two by an edge, the graph obtained from a gear graph by duplicating each vertex of degree three by an edge and the graph obtained by duplication of all rim vertices of a gear graph by an edge when $k \leq 3$ is face product cordial and total face product cordial graph.

Keywords: Book Graph, Duplication, Face product cordial labeling, Gear graph, Product cordial labeling, Total face product cordial labeling.

AMS Subject Classification (2020): 05C78, 05C76.

1. Introduction

A graph labeling is the assignment of integers to the vertices or edges or both subject to conditions. For different graph labeling techniques we use a dynamic survey of graph labeling by Gallian [1]. We start with a simple, finite, undirected graph $G = (V(G), E(G))$ with vertex set V and edge set E of G. For different notation and terminology we follow Gross and Yellen [2]. Face Cordial labeling and Total face product cordial labeling were given by P. Lawrence Rozario Raj and R. Lawrence Joseph Manoharan [3]. Now we provide brief summary of definitions and other information which are necessary for the present investigations.

2. Definitions

Definition 2.1 A cycle in a graph is a non-empty trail in which only the first and last vertices are equal.

Definition 2.2 The graph $W_n = C_n + K_1$ is called a wheel graph. The vertex corresponding to K_1 is called the apex vertex and the vertices corresponding to Cn are called the rim vertices [1].

Definition 2.3 A book graph $B(m, n)$ is a graph obtained by identifying n edges taking one edge from each of the n distinct copies of C_m, where n is called number of pages of the book $B(m, n)$. The edge obtained by identifying n edges is called the spine or base of the graph $B(m, n)$ [4].

Definition 2.4 A gear graph G_n is obtained from the wheel graph W_n by adding a vertex between every pair of adjacent vertices in the cycle C_n [5].

Definition 2.5 A mapping f ꞉ V (G) → {0, 1} is called a binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. We denote $v_f(0)$ as the number of vertices with label 0 and $v_f(1)$ as the number of vertices with label 1 [5].

Definition 2.6 A product cordial labeling of graph G with vertex set V is a function f ꞉ V (G) → {0, 1} such that each edge uv is assigned the label f(u)f(v), the number of vertices with label 0 and the number of vertices with label 1 differ by atmost 1 and the number of edge with label 0 and the number of edge with label 1 differ by atmost 1. A graph which admits product cordial labeling is called a product cordial graph. Sundaram, Ponraj and Somasundaram [6] introduced a product cordial labeling.

Definition 2.7 The neighbourhood of a vertex v of a graph is the set of all vertices adjacent to v. It is denoted by N(v) [5].

Definition 2.8 Duplication of a vertex of the graph G is the graph G' obtained from G by adding a new vertex v' to such that $N(v') = N(v)$ [5].

Definition 2.9 Duplication of a vertex v_k by a new edge $e = v_k, v'_k$ in a graph G produces a new graph G' such that $N(v'_k) = \{v_k, v''_k\}$ and $N(v''_k) = \{v_k, v'_k\}$. The symbols of duplication of a vertex by a new edge and duplication of an edge by a new vertex were found by Vaidya and Bansara [7].

Definition 2.10 For a planar graph G, the binary vertex labeling function is defined as g ꞉ V (G) → {0, 1} and g(v) is the label of the vertex v of G under g, induced edge labeling function g^*꞉ E(G) → {0, 1} is given as if e = uv then $g^*(uv) = g(u)g(v)$ and induced face labeling function g^{**} ꞉ F(G) → {0, 1} is given as if $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_m$ are the vertices and edges of face f then $g^{**}(f) = g(v_1)g(v_2) \ldots g(v_n)g^*(e_1)g^*(e_2) \ldots g^*(e_m)$. Let us denote $v_g(i)$ is the number of vertices of G having label i under g, $e_g(i)$ is the number of edges of G having label i under g^* and $f_g(i)$ is the number of interior faces of G having label i under g^{**} for i = 1, 2. g is called the face product cordial labeling of graph G if if $|v_g(0) - v_g(1)| \leq 1$, $|e_g(0) - e_g(1)| \leq 1$ and $|F_g(0) - F_g(1)| \leq 1$. A graph G is a face product cordial graph if it admits face product cordial labeling [3]. P. Lawrence Rozario Raj and R. Lawrence Joseph Manoharan [3] introduced a face product cordial labeling.

Definition 2.11 For a planar graph G, the binary vertex labeling function is defined as g ꞉ V (G) → {0, 1} and g(v) is the label of the vertex v of G under g, induced edge labeling function g^*꞉ E(G) → {0, 1} is given as if e = uv then $g^*(uv) = g(u)g(v)$ and induced face labeling function g^{**} ꞉ F(G) → {0, 1} is given as if $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_m$ are the vertices and edges of faces f then $g^{**}(f) = g(v_1)g(v_2) \ldots g(v_n)g^*(e_1)g^*(e_2) \ldots g^*(e_m)$. Let g(0) and g(1) be the sum of the number of vertices, edges and interior faces having labels 0 and labels 1 respectively. g is called total face product cordial labeling of graph G if $|g(0) - g(1)| \leq 1$. A graph G is called total face product cordial graph if it admits total face product cordial labeling [3]. P. Lawrence Rozario Raj and R. Lawrence Joseph Manoharan [3] introduced a total face product cordial labeling.

Note:- Throughout the article we use i ∈ $[k]$, whenever $1 \leq i \leq k$.

3. Results

Theorem 3.1 A book graph $B(m, k)$ is face product cordial graph if both m and k are odd and $m \geq 3$.
Proof: Let $B(m, k)$ be the book graph with both m and k are odd with $m \geq 2$ is obtained by identifying k edges taking one edge from each of the k distinct copies of C_m, where k is called number of the pages of $B(m, k)$. The edge obtained by identifying k edges is called the spine. Thus, $|V (B(m, k))| = (m - 2)k + 2, |E(B(m, k))| = (m - 1)k + 1$ akd $|F(B(m, k))| = k$. Name the vertices of G as follows: v_0 and v'_0 to the end vertices of the spine in $B(m, k)$. Let $v_{11}, v_{21}, v_{31}, \ldots, v_{(m-2)n}$ be the consecutive vertices of 1st copy of C_m, 2nd copy of C_m, ..., k^{th} copy of C_m other than the end vertices of the spine.
Define $\mathfrak{f}$꞉ $V (B(m, k)) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1, & \text{if } x \in \{v_0, v'_0\}; \\ 1, & \text{if } x = v_{ij}, \quad i \in [\dfrac{m-1}{2}], j \in [\dfrac{k-1}{2}]; \\ 1, & \text{if } x = v_{ij}, i = 1, j \in [\dfrac{k+1}{2}]; \\ 0, & otherwise. \end{cases}$$

In view of above labeling pattern we have, $v_f(1) = \dfrac{(m-2)n+3}{2}, v_f(0) = \dfrac{(m-2)n+1}{2}, e_f(1) = \dfrac{(m-1)n}{2}, e_f(0) = \dfrac{(m-2)n+1}{2}, F_f(1) = \dfrac{n-1}{2}$ akd $F_f(0) = \dfrac{n+1}{2}$. Thus, $|e_f(0) - e_f(1)| \le 1, |v_f(0) - v_f(1)| \le 1$ akd $|F_f(0) - F_f(1)| \le 1$. Hence, B(m, n) is a face product cordial graph when m and n both odd with m $\ge$ 3 as it admits face product cordial labeling.

Illustration 3.1: Face product cordial labeling of $B(5,3)$ is shown in the Figure 1.

Figure 1: Face product cordial labeling of $B(5,3)$

Theorem 3.2 Gear graph Gn is a face product cordial and total face product cordial graph for n $\ge$ 3 for odd values of n.

Proof: Let W_n be a wheel graph with the apex vertex v_0 and consecutive rim vertices as $v_1, v_2, \dots, v_n$. The gear graph G_n is obtained by subdividing each of the rim edges $v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1$ of W_n by the vertices $u_1, u_2, \dots, u_n$ respectively. Thus, $|V(G_n)| = 2k + 1, |E(G_n)| = 3k$ akd $|F(G_n)| = k$.

Define a function $f : V(G) \rightarrow \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1, & \text{if } x = v_0; \\ 1, & \text{if } x = u_i, i \in [\dfrac{k-1}{2}]; \\ 1, & \text{if } x = v_i, i \in [\dfrac{k+1}{2}]; \\ 0, & otherwise. \end{cases}$$

In view of above labeling pattern we have, $v_f(0) = k, v_f(1) = k + 1, e_f(0) = \dfrac{3n+1}{2}, e_f(1) = \dfrac{3n-1}{2}, F_f(0) = \dfrac{n+1}{2}, F_f(1) = \dfrac{n-1}{2}$. Thus, $|e_f(0) - e_f(1)| \le 1, |v_f(0) - v_f(1)| \le 1$ akd $|F_f(0) - F_f(1)| \le 1$. Hence, G is face product cordial graph for all odd k and $k \ge 3$. Also, $g(0) = v_f(0) + e_f(0) + F_f(0) = k + \dfrac{3n+1}{2} + \dfrac{n+1}{2} = 3k + 1$ and $g(1) = v_f(1) + e_f(1) + F_f(1) = (k + 1) + \dfrac{3n-1}{2} + \dfrac{n-1}{2} = 3k$. Thus, $|g(0) - g(1)| \le 1$. Hence, it is total face product cordial graph.

Illustration 3.2: Face product cordial labeling and total face product cordial labeling of G_3 is shown in the Figure 2.
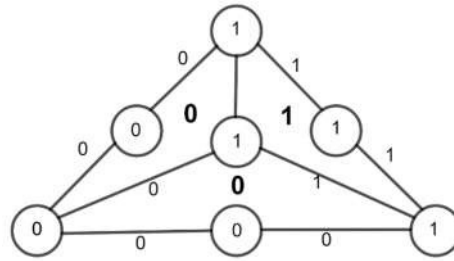
Figure 2: Face prodct cordial labeling of G_3 .

Theorem 3.3 The graph obtained by duplication of each vertex of degree two by an edge in G_n admits face product cordial labeling and total face product cordial labeling both for $k \geq 3$.

Proof: Let W_n be a wheel graph with the apex vertex v_0 and consecutive rim vertices as $v_1, v_2, \ldots, v_n$. The gear graph G_n is obtained by subdividing each of the rim edges $v_1v_2, v_2v_3, \ldots, v_{n-1}v_n, v_nv_1$ of W_n by the vertices $u_1, u_2, \ldots, u_n$ respectively. Thus, $|V(G_n)| = 2k + 1$, $|E(G_n)| = 3k$ akd $|F(G_n)| = k$. Let G be the graph obtained from G_n by duplicating each vertex u_i of degree two by an edge $u'_iu''_i$ respectively for all i = 1, 2, 3, \ldots, k. Thus, $|V(G)| = 4k + 1$, $|E(G)| = 6k$ akd $|F(G)| = 2k$.

Define a function $f : V(G) \rightarrow \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1, & \text{if } x = v_0; \\ 1, & \text{if } x = v_i, i \in [k]; \\ 1, & \text{if } x = u_i, i \in [k]; \\ 0, & \text{if } x \in \{u'_iu''_i\}, i \in [k]. \end{cases}$$

In view of above labeling pattern we have, $v_f(0) = 2k, v_f(1) = 2k + 1, e_f(0) = 3k, e_f(1) = 3k, F_f(0) = k, F_f(1) = k$. Thus, $|e_f(0) - e_f(1)| \leq 1, |v_f(0) - v_f(1)| \leq 1$ akd $|F_f(0) - F_f(1)| \leq 1$. Hence, G_n is face product cordial graph for all odd k and $k \geq 3$. Also, $g(0) = v_f(0) + e_f(0) + F_f(0) = 2k + 3k + k = 6k$ and $g(1) = v_f(1) + e_f(1) + F_f(1) = (2k + 1) + 3k + k = 6k + 1$. Thus, $|g(0) - g(1)| \leq 1$. Hence, it is total face product cordial graph.

Illustration 3.3: Face product cordial labeling and total face product cordial labeling of G_5 is shown in the Figure 3.
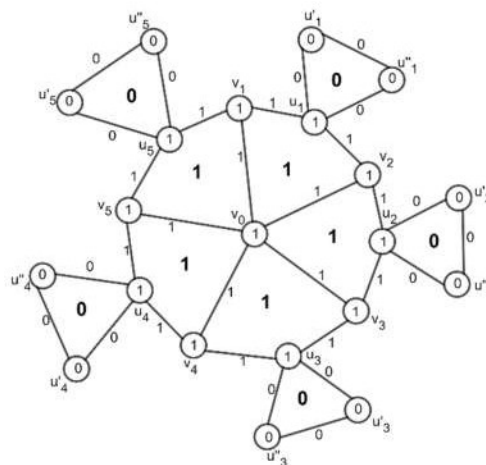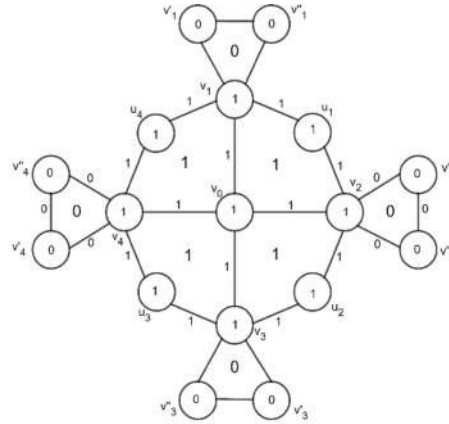

Figure 3: Face prodct cordial labeling of G_5 .

Theorem 3.4 The graph obtained by duplication of each vertex of degree three by an edge in G_n is both face product cordial graph and total face product cordial graph.

Proof: Let W_n be a wheel graph with the apex vertex v_0 and consecutive rim vertices as v_1, v_2, ... , v_n. The gear graph G_n is obtained by subdividing each of the rim edges $v_1v_2, v_2v_3, \ldots, v_{n-1}v_n, v_nv_1$ of W_n by the vertices $u_1, u_2, \ldots, u_n$ respectively. Thus, $|V(G_n)| = 2k + 1$, $|E(G_n)| = 3k$ akd $|F(G_n)| = k$. Let G be the graph obtained from G_n by duplicating each vertex v_i of degree three by an edge $v'_iv''_i$ respectively for all $i = 1, 2, 3, \ldots, k$. Thus, $|V(G)| = 4k + 1, |E(G)| = 6k$ akd $|F(G)| = 2k$.

Define a function $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1, & \text{if } x = v_0; \\ 1, & \text{if } x = v_i, i \in [k]; \\ 1, & \text{if } x = u''_i, i \in [k]; \\ 0, & \text{if } x \in \{v'_iv''_i\}, i \in [k]. \end{cases}$$

In view of above labeling pattern we have, $v_f(0) = 2k, v_f(1) = 2k + 1, e_f(0) = 3k, e_f(1) = 3k, F_f(0) = k, F_f(1) = k$. Thus, $|e_f(0) - e_f(1)| \leq 1, |v_f(0) - v_f(1)| \leq 1$ akd $|F_f(0) - F_f(1)| \leq 1$. Hence, G_n is face product cordial graph for all odd k and $k \geq 3$. Also, $g(0) = v_f(0) + e_f(0) + F_f(0) = 2k + 3k + k = 6k$ and $g(1) = v_f(1) + e_f(1) + F_f(1) = (2k + 1) + 3k + k = 6k + 1$. Thus, $|g(0) - g(1)| \leq 1$. Hence, it is total face product cordial graph.

Illustration 3.4: Face product cordial labeling and total face product cordial labeling of G_4 is shown in the Figure 4.

Figure 4: Face prodct cordial labeling of G_4

Theorem *3.5* The graph obtain by duplication of each of the rim vertices in G_n is both face product cordial graph and total face product cordial graph if k is even.

Proof: Let W_n be a wheel graph with the apex vertex v_0 and consecutive rim vertices as v_1, v_2, ... , v_n. The gear graph G_n is obtained by subdividing each of the rim edges $v_1v_2, v_2v_3, \ldots, v_{n-1}v_n, v_nv_1$ of W_n by the vertices $u_1, u_2, \ldots, u_n$ respectively. Thus, $|V(G_n)| = 2k + 1$, $|E(G_n)| = 3k$ akd $|F(G_n)| = k$. Let G be the graph obtained from G_n by duplicating each vertex v_i and u_i by an edge $v'_iv''_i$ and $u'_iu''_i$ respectively for all $i = 1, 2, 3, \ldots, k$. Thus, $|V(G)| = 6k + 1, |E(G)| = 9k$ akd $|F(G)| = 3k$.

Case 1: $n \equiv 0 \pmod 4$.
Define a function $f : V(G) \to \{0, 1\}$ as follows:

$$f(x) = \begin{cases} 1, & \text{if } x = v_i, i \in \{0, [k]\}; \\ 1, & \text{if } x = u_i, i \in [k]; \\ 1, & \text{if } x = v'_i, i \in [\frac{k}{4}]; \\ 1, & \text{if } x = u'_i, i \in [\frac{k}{4}]; \\ 1, & \text{if } x = u''_i, i \in [\frac{k}{4}]; \\ 0, & \text{otherwise.} \end{cases}$$

In view of above labeling pattern we have, $v_f(0) = 3k, v_f(1) = 3k+1, e_f(0) = 9n, e_f(1) = 9n, F_f(0) = 3n, F_f(1) = 3n$. Thus, $|e_f(0) - e_f(1)| \leq 1, |v_f(0) - v_f(1)| \leq 1$ and $|F_f(0) - F_f(1)| \leq 1$. Hence, G^2 is face product cordial graph for all odd k and $k \geq 3$. Also, $g(0) = v_f(0) + e_f(0) + F_f(0) = 3k + 9n + \frac{3n}{2} = 9k$ and $g(1) = v_f(1) + e_f(1) + F_f(1) = (3k+1) + \frac{9n}{2} + \frac{3n}{2} = 9k+1$. Thus, $|g(0) - g(1)| \leq 1$. Hence, it is total face product cordial graph.

Case 2: n ≡ 1(mod 4).
Define a function $f : V(G) \to \{0, 1\}$ such that,

$$f(x) = \begin{cases} 1, & \text{if } x = v_i, i \in \{0, [k]\}; \\ 1, & \text{if } x = u_i, i \in [k]; \\ 1, & \text{if } x = v'_i, i \in [\frac{k+3}{4}]; \\ 1, & \text{if } x = v''_i, i \in [\frac{k-1}{4}]; \\ 1, & \text{if } x = u'_i, i \in [\frac{k-1}{4}]; \\ 1, & \text{if } x = u''_i, i \in [\frac{k-1}{4}]; \\ 0, & otherwise. \end{cases}$$

In view of above labeling pattern we have, $v_f(0) = 3k, v_f(1) = 3k+1, e_f(0) = [\frac{9n+1}{2}], e_f(1) = [\frac{9n-1}{2}], F_f(0) = [\frac{3n+1}{2}], F_f(1) = [\frac{3n-1}{2}]$. Thus, $|e_f(0) - e_f(1)| \leq 1, |v_f(0) - v_f(1)| \leq 1$ and $|F_f(0) - F_f(1)| \leq 1$. Hence, G_n is face product cordial graph for all odd k and $k \geq 3$. Also, $g(0) = v_f(0) + e_f(0) + F_f(0) = 3k + [\frac{9n+1}{2}] + [\frac{3n+1}{2}] = 9k+1$ and $g(1) = v_f(1) + e_f(1) + F_f(1) = (3k+1) + [\frac{9n-1}{2}] + [\frac{3n-1}{2}] = 9k$. Thus, $|g(0) - g(1)| \leq 1$. Hence, it is total face product cordial graph.

Case 3: n ≡ 2(mod 4).
Define a function $f : V(G) \to \{0, 1\}$ such that,

$$f(x) = \begin{cases} 1, & \text{if } x = v_i, i \in \{0, [k]\}; \\ 1, & \text{if } x = u_i, i \in [k]; \\ 1, & \text{if } x = v'_i, i \in [\frac{k+2}{4}]; \\ 1, & \text{if } x = v''_i, i \in [\frac{k-2}{4}]; \\ 1, & \text{if } x = u'_i, i \in [\frac{k-2}{4}]; \\ 1, & \text{if } x = u''_i, i \in [\frac{k-2}{4}]; \\ 0, & otherwise. \end{cases}$$

In view of above labeling pattern we have, $v_f(0) = 3k, v_f(1) = 3k+1, e_f(0) = 9n, e_f(1) = 9n, F_f(0) = 3n, F_f(1) = 3n$. Thus, $|e_f(0) - e_f(1)| \leq 1, |v_f(0) - v_f(1)| \leq 1$ and $|F_f(0) - F_f(1)| \leq 1$. Hence, G^2 is face product cordial graph for all odd k and $k \geq 3$. Also, $g(0) = v_f(0) + e_f(0) + F_f(0) = 3k + 9n + \frac{3n}{2} = 9k$ and $g(1) = v_f(1) + e_f(1) + F_f(1) = (3k+1) + \frac{9n}{2} + \frac{3n}{2} = 9k+1$. Thus, $|g(0) - g(1)| \leq 1$. Hence, it is total face product cordial graph.

Case 4: n ≡ 3(mod 4).
Define a function $f : V(G) \to \{0, 1\}$ such that,

$$f(x) = \begin{cases} 1, & \text{if } x = v_i, i \in \{0, [k]\}; \\ 1, & \text{if } x = u_i, i \in [k]; \\ 1, & \text{if } x = v'_i, i \in [\dfrac{k+1}{4}]; \\ 1, & \text{if } x = v''_i, i \in [\dfrac{k+1}{4}]; \\ 1, & \text{if } x = u'_i, i \in [\dfrac{k+1}{4}]; \\ 1, & \text{if } x = u''_i, i \in [\dfrac{k+3}{4}]; \\ 0, & otherwise. \end{cases}$$

In view of above labeling pattern we have, $v_f(0) = 3k, v_f(1) = 3k+1, e_f(0) =]\dfrac{9n+1}{2}[, e_f(1) =]\dfrac{9n-1}{2}[, F_f(0) =]\dfrac{3n+1}{2}[, F_f(1) =]\dfrac{3n-1}{2}[$. Thus, $|e_f(0) - e_f(1)| \leq 1, |v_f(0) - v_f(1)| \leq 1$ and $|F_f(0) - F_f(1)| \leq 1$. Hence, G_n is face product cordial graph for all odd k and $k \geq 3$. Also, $g(0) = v_f(0) + e_f(0) + F_f(0) = 3k +]\dfrac{9n+1}{2}[+]\dfrac{3n+1}{2}[= 9k+1$ and $g(1) = v_f(1) + e_f(1) + F_f(1) = (3k+1) +]\dfrac{9n+1}{2}[+]\dfrac{3n-1}{2}[= 9k$. Thus, $|g(0) - g(1)| \leq 1$. Hence, it is total face product cordial graph.

Thus, by all the above cases it satisfies face and total face product cordial labeling if n is even. Hence the graph obtain by duplication of each rim vertices in G_n admits both face and total face product cordial labeling.

Illustration 3.4: Face product cordial labeling and total face product cordial labeling of G_8 is shown in the Figure 5.
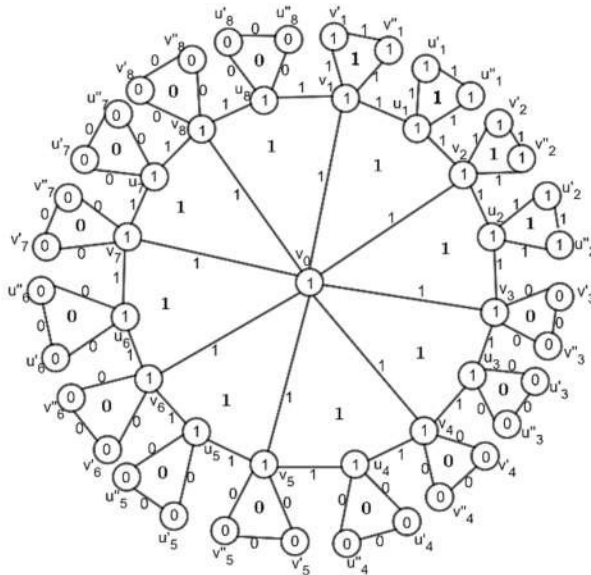

Figure 5: Face product cordial labeling G_8

4. Conclusion

We applied Face product cordial labeling on Book graph when m and n both odd with $m \geq 3$. Also we applied Face product cordial labeling and Total Face product cordial labeling on Gear graph ($k \geq 3$). We also showed three results on the graph obtained by switching of a vertex of distinct degrees in the gear graph is Face and Total face cordial labeling under the condition said erlier.

References

[1] Gallian, J. A., (2021) A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, #DS6.

[2] Gross, J., and Yellen, J., Handbook of Graph Theory, CRC Press.

[3] Lawrence Rozario Raj P. and Lawrence Joseph Manoharan R., (2015): Face and Total Face Product Cordial Labeling of Graphs, IJISET - International Journal of Innovative Science, Engineering & Technology, (2), 2348-7968.

[4] Prajapati, U. M., and Raval, K. K., (2017): Some Graphs in the Context of Product Cordial Labeling, Mathematics Today (33), 58-66.

[5] Prajapati, U. M., and Raval, K. K., (2016): Product Cordial Graph in the Context of Some Graph Operations on Gear Graph, Open Journal of Discrete Mathematics, (6), 259-267.

[6] Sundaram, R. P. M., and Somsundaram, S., (2004): Product Cordial Labeling of Graphs, Bulletin of Pure and Applied Science (Mathematics & Statistics), (23E), 155-163.

[7] Vaidya, S. K., and Bansara, C. M., (2011): Product Cordial Graphs in the Context of Some Graph Operations, International Journal of Mathematics and Scientific Computing, (1), 1-6.