Generating Dio-3 Triples using the Second-Order Polynomials with Incisive Properties

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Abstract: *In this communication, we achieve special Diophantine triples involving second-order polynomials, where the product of any two members of the set subtracted by their sum and increased by integer-coefficient polynomial yields a perfect square. Also provides graphical representation of the Dio-3 Triples using MATLAB.*

Keywords: Special Diophantine Triples, Perfect Square, Star Number.

1. INTRODUCTION

The enormous numbers of unresolved issues in number theory that appear to be solvable from the outside make it attractive. Unsolved issues in number theory are unsolved for a reason, of course. Although they appear to be simple, numbers have a remarkably complex structure that we only partially comprehend [9-13]. Diophantus researched the feature that the product of any two of their separate components is one less than a square has a very long history. If $x_i.x_j + n$ is a perfect square for every $1 \le i < j \le s$, then a collection of s unique non-null integers $(g_1, g_2, \ldots g_s)$ is referred to as a Dio s-tuple with attributes $D(n)$ .A number of mathematicians explored the existence of Diophantine triples with the property $D(n)$ for any integer n and, moreover, for any linear polynomial in n . One might now recommend a thorough examination of many topics relating to Diophantine triples [1–8].

In this study, we provide unique Diophantine triples (a,b,c) involving polynomials, where the product of any two members of the set, subtracted by their sum, and increased by an integer-coefficient polynomial is a perfect square. Also provides graphical representation of the Dio-3 Triples using MATLAB.

2. NOTATION

$star_n$: Star number of rank $n = 6\text{n}(\text{n} -1) + 1$

3. BASIC DEFINITION

A set of three different second order polynomial with integer coefficients (a_1, a_2, a_3) is said to be a special Diophantine triple with property $D(n)$if $\text{a}_\text{i} * a_j - (\text{a}_\text{i} + a_j) + n$ is a perfect square for all $1 \le i < j \le 3,$ where n may be non-zero integer or polynomial with integer coefficients.

4. ANALYTICAL APPROACH

4.1. Development of the distinctive dio-3 triples using the second order polynomial $6n^2 - 6n + 1$ and $6n^2 - 18n + 1$

Let $a = 6n^2 - 6n + 1$ and $b = 6n^2 - 18n + 1$

$$ab - (a+b) + 72n + 10 = 36n^4 - 144n^3 + 108n^2 + 72n + 9$$
$$= (6n^2 - 12n - 3)^2$$
$$= \lambda^2 \quad (1)$$

Equation (1) is a perfect square.

$ab - (a+b) + 72n + 10 = \lambda^2$ where $\lambda = 6n^2 - 12n - 3$

Allowing c to be a non-zero integer,

$$ac - (a+c) + 72n + 10 = \mu^2 \quad (2)$$
$$bc - (b+c) + 72n + 10 = \omega^2 \quad (3)$$

Solving (2) and (3) one may get

$$(a-b) + (b-a)(72n+10) = (b-1)\mu^2 - (a-1)\omega^2 \quad (4)$$

Setting $\mu = y + (a-1)T$ and $\omega = y + (b-1)T$
(5)

Applying Equation (5) in (4) one may get

$$y^2 = (b-1)(a-1)T^2 + 72n + 9$$
(6)

Initial solution of (6) is given by,

$y_0 = (6n^2 - 12n - 3)$ and $T_0 = 1$

Since $\mu = y + (a-1)T$ and $\omega = y + (b-1)T$, we obtain that,

$\mu = 12n^2 - 18n - 3$

Therefore, the equation (2) becomes,

$$ac - c - a + 72n + 10 = \mu^2$$
$$\Rightarrow (6n^2 - 6n)c = 144n^4 - 432n^3 + 258n^2 + 30n$$
$$\Rightarrow c = 24n^2 - 48n - 5$$

Hence, the triples $(a,b,c) = (6n^2 - 6n + 1, 6n^2 - 18n + 1, 24n^2 - 48n - 5)$ are Diophantine triples with the property $D(72n + 10)$.

The following table provides some numerical illustrations.

Table 1

n	Diophantine Triples	$D(72n+10)$
1	(1,–11,–29)	82
2	(13,– **1**,–5)	154
3	(37,1,67)	226
4	(73,25,187)	298

Remarkable Observation:

It is noted that, the above second order polynomial is of the form $(a,b,c) = (star_n, star_{n-1} - 12, 4star_{n-2} + 72n - 153)$. Also all the triples are odd with even number attributes.

4.2. Development of the distinctive dio-3 triples using the second order polynomial $6n^2 - 6n + 1$ and $6n^2 - 30n + 31$

Let $a = 6n^2 - 6n + 1$ and $b = 6n^2 - 30n + 31$

$$ab - (a + b) - 36n + 145 = 36n^4 - 288n^3 + 720n^2 - 576n + 144$$
$$= (6n^2 - 24n + 12)^2$$
$$= \lambda^2 \quad (7)$$

Equation (7) is a perfect square.

$ab - (a + b) - 36n + 145 = \lambda^2$ where $\lambda = 6n^2 - 24n + 12$

Allowing c to be a non-zero integer,

$$ac - (a + c) - 36n + 145 = \mu^2 \quad (8)$$

$$bc - (b + c) - 36n + 145 = \omega^2 \quad (9)$$

Solving (8) and (9) one may get

$$(a - b) + (b - a)(-36n + 145) = (b - 1)\mu^2 - (a - 1)\omega^2 \quad (10)$$

Setting $\mu = y + (a - 1)T$ and $\omega = y + (b - 1)T$
(11)

Applying Equation(11) in (10) one may get

$$y^2 = (b-1)(a-1)T^2 - 36n + 145 \tag{12}$$

Initial solution of (12) is given by,

$y_0 = (6n^2 - 24n + 12)$ and $T_0 = 1$

Since $\mu = y + (a-1)T$ and $\omega = y + (b-1)T$, we obtain that,

$\mu = 12n^2 - 42n + 12$

Therefore, the equation (7) becomes,

$$ac - c - a - 36n + 145 = \mu^2$$
$$\Rightarrow (6n^2 - 18n)c = 144n^4 - 1008n^3 + 2058n^2 - 990n$$
$$\Rightarrow c = 24n^2 - 96n + 55$$

Hence, the triples $(a,b,c) = (6n^2 - 6n + 1\,,\, 6n^2 - 30n + 31, 24n^2 - 96n + 55)$ are Diophantine triples with the property $D(-36n + 145)$.

The following table provides some numerical illustrations

Table 2

n	**Diophantine Triples**	$D(-36n+145)$
1	$(-11,7,-17)$	109
2	$(-11,-5,-41)$	73
3	$(1,-5,-17)$	37
4	$(25,7,55)$	1

Remarkable Observation:

It is noted that, the above second order polynomial is of the form $(a,b,\ c) = (star_n\ ,\ star_{n-2} - 6,\ 4star_{n-3} + 72n - 237)$. Also all the triples and their attributes are odd.

4.3. Development of the distinctive dio-3 triples using the second order polynomial $6n^2 - 5$ and $6n^2 - 12n + 13$

Let $a_n = 6n^2 - 5$ and $a_{n-1} = 6n^2 - 12n + 13$

$$\begin{aligned} a_n\, a_{n-1} + 30 &= 36n^4 - 72n^3 - 24n^2 + 60n + 25 \\ &= (6n^2 - 6n - 5)^2 \\ &= \lambda^2 \end{aligned} \tag{13}$$

Equation (7) is a perfect square.

$a_n a_{n-1} + 30 = \lambda^2$ where $\lambda = 6n^2 - 6n - 5$

Allowing c to be a non-zero integer,

$$a_n\, c + 30 = \mu^2 \qquad (14)$$

$$a_{n-1}\, c + 30 = \omega^2 \qquad (15)$$

Solving (14) and (15) one may get

$$(a_n - a_{n-1})c = \mu^2 - \omega^2 \qquad (16)$$

Setting $\mu = a_n + \lambda$ and $\omega = a_{n-1} + \lambda$ (17)

Applying Equation(17) in (16) one may get

$$c = a_n + a_{n-1} + 2\lambda$$
$$= 24n^2 - 24n - 14 \qquad (18)$$

Hence, the triples $(a_n, a_{n-1}, c) = (6n^2 - 5,\ 6n^2 - 12n + 13,\ 24n^2 - 24n - 14)$ are Diophantine triples with the property $D(30)$.

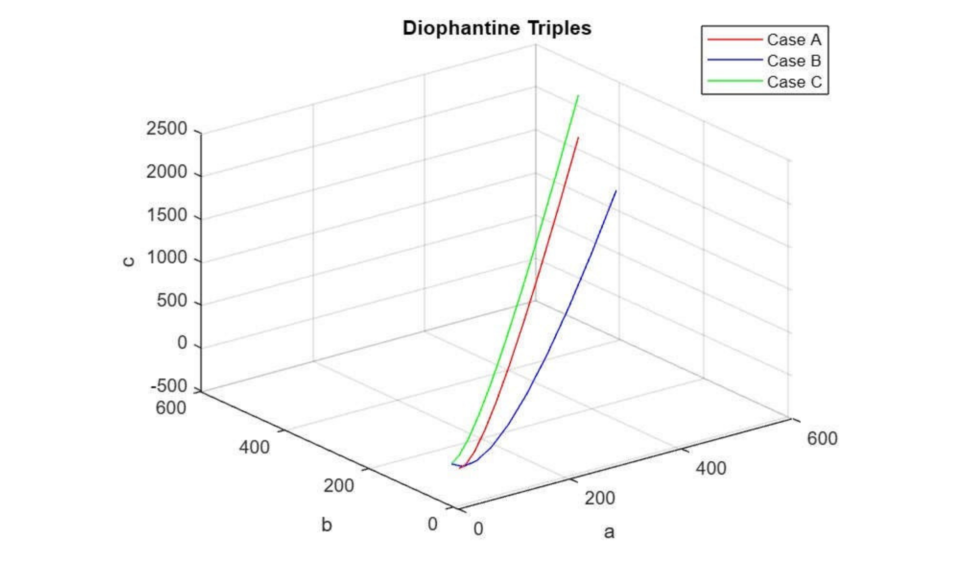
The following table provides some numerical illustrations

Table 3

n	**Diophantine Triples**	$D(30)$
1	$(1,-5,-14)$	30
2	$(19,1,34)$	30
3	$(49,19,130)$	30
4	$(91,49,274)$	30

Remarkable Observation:

It is noted that, the above second order polynomial is of the form $(a_n, a_{n-1}, c) = (star_n + 6n - 6, star_{n-1} + 6n, 4star_{n-2} + 96n - 162)$



5. CONCLUSION

In this article, we have shown a few instances of how to build unique Dio 3 tuples involving the second order polynomial with the right attributes. Also provides graphical representation of the Dio-3 Triples using MATLAB. In conclusion, one can look for Dio 3 tuples for various polynomials with their corresponding attributes.

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