The Notion of new mappings in Minimal Structure

R. Buvaneswari, m.s.shakin banu, D.Ragavi

PG and Research Department of mathematics Cauvery college for women, Trichy-18 Tamil Nadu, India

Abstract— This paper aims forming at the some mappings like m_X -feebly regular open with its complement mapping. These concepts are defined at the m_X -feebly regular continuous function and also discussed at the some related theorems in it.

Keywords— m_X -feebly open, m_X -feebly closed, m_X -feebly interior, m_X -feebly closure, m_X -feebly clopen, m_X -feebly regular open and m_X -feebly regular closed.

I INTRODUCTION

General topology is the main role of mathematical field. In 1963, Levine introduced at the concepts of semi open set and semi-continuous. The semi open sets, preopen sets, α -open sets, β -open sets, b-open sets and δ -open sets play an important role in the research of generalization of continuity in topological spaces. By using these sets several authors introduced at the various types of Non-continuous functions. Further the analogy in their definitions and properties suggests the need of formulating in the setting of functions. In 1982 Tong., J investigated at the separation axioms and decomposition of continuity. In 1982, S.N Maheswari and P.C. Jain defined and studied at the concepts of feebly open and feebly closed sets in topological spaces. In 2000, the concepts of minimal structure (briefly m_X -structure) was introduced by V. Popa and T. Noiri. They introduced at the notions of m_X-open sets and m_X -closed sets and characterize of those sets using m_X -closure and m_X operators, respectively and also obtained the definitions and characterizations of some mappings by using at the concept of minimal structure.

II PRELIMINARIES

Definition 2.1: Let (X,τ) be a topological space. A subset A of X is said to be

1) α-open [9] if A $\subset$ int (cl(int(A)))
2) Semi-open [6] if A $\subset$ cl (int (A))
3) Preopen [9] if A $\subset$ int (cl (A))
4) b-open [2] if A $\subset$ int (cl (A)) $\cup$ cl (int (A))
5) β -open [1] or semi-preopen if A $\subset$ cl (int (cl (A)))
6) Feebly open [7] if A $\subset$ s cl (int (A))
7) Feebly closed [7] if int (cl (A)) $\subset$ A

The family of all α -open (resp., semi-open, preopen, b-open, β -open, feebly open, feebly closed) sets in (X,τ) is denoted by α (X) (resp., $SO(X), PO(X), BO(X), \beta(X), FO(X)$.

Definition 2.2 [11, 12]: A subfamily m_X of the power set P(X) of a non-empty set X is called a minimal structure (briefly m-structure) on X if $\varphi \in m_X$ and $X \in m_X$. By (X, m_X) we denote a non-empty set X with a minimal structure m_X on X and call it an m-space. Each member of m_X is said to be m_X -open and the complement of an m_X -open is said to be m_X -closed.

Remark 2.3: Let (X,τ) be a topological space. Then the families $\tau, SO(X), PO(X), BO(X) and \beta(X)$ are all m-structure on X.

Definition 2.4: Let (X, m_X) be a m –space. For a subset A of m_X –closure of A and m_X –interior of A are defined in [8] as follows:

(i) m_X–cl (A) $= \cap\{F : A \subset F, X - F \in m_X\}$
(ii) m_X–int (A) $= \cup\{U : U \subset A, U \in m_X\}$

Remark 2.5: Let (X, τ) be a topological space and let A be a subset of X. If $m_X = \tau$ (resp., $so(X), PO(X), \alpha(X), BO(X)$ and $\beta(X)$ then we have

(a) m_X–cl (A)= cl (A) (resp s.cl (A), P. cl (A), α.cl (A), b.cl (A) and β .cl (A)).
(b) m_X–int (A)= int (A) (resp s .int (A),P.int (A), α.int (A), b . int (A) and β .int (A)).

Remark 2.6: Let A be a subset of (X, m_X)

(a) The union of all m_X –semi open sets of X contained in A is called the m_X –semi-interior of A.
(b) The intersection of all m_X –semi closed sets of X containing A is called the m_X –semi closure of A.
(c) A is called m_X –semi preopen if A $\subset m_X$ –cl (m_X –int(m_X –cl(A))). Its complement is m_X –semi-preclosed.

Definition 2.7 [7]: A subset A of (X, m_X) is said to be m_X – regular open (briefly m_X –RO) if A= m_X –int (A) and m_X –regular closed if A= m_X –cl (A).

Definition 2.8 [12]: Let $f : (X, m_X) \to (Y, m_Y)$ be a function between a space (X, m_X) with minimal structure m_X and a topological space (Y, m_Y). Then *f* said to be *m*-continuous if for each *x* and each open set V containing *f(x)*, there exists an m-open set U containing *x* such that *f*(U) $\subseteq$ V.

Definition 2.9 [5]: Let (X, m_X) be a topological space. Any subset *A* of *X* is called feebly clopen if it is both feebly open and feebly closed.

Definition 2.10: A map $f : X \to Y$ is said to be

1. Feebly closed (resp., feebly open) [10] if the image of each closed set (resp., open set) in X is feebly closed (resp., feebly open) set in Y
2. Feebly continuous [10] if f^{-1}(V) is feebly open in X for each set V of Y.
3. Feebly clopen [5] if the image of every open and closed set in X is both feebly open and closed in Y.

Definition2.11[3]:A subset (E, m_X) of a m_X –space (X, m_X) is said to be m_X –feebly open if E $\subset m_X$ –s cl (m_X –int(E)).

Definition2.12[3]: A subset (E, m_X) of a m_X–space (X, m_X) is said to be m_X –feebly closed if m_X –int(m_X –cl(E)) $\subset$E.

Definition2.13[3]: The m_X –feebly closure of (E, m_X) is the intersection of all m_X –feebly closed set containing (E, m_X) and is denoted by m_X –*f*.cl(E).

Definition2.14[3]: The m_X –feebly interior of (E, m_X) is the union of all m_X –feebly open sets contained in (E, m_X) and is denoted
by m_X –*f*.int(E).

Definition2.15[13]:A subset A of (X, m_X) is said to be m_X – **Feebly regular open** (briefly m_X –*f*.reg.open) if A = m_X–*f*.int (m_X –*f*.cl (A)).

Definition2.16[13]:A subset A of (X, m_X) is said to be m_X – **Feebly regular closed** if A = m_X –*f*.cl (m_X – *f*.int (A)) (briefly m_X –*f*.reg.closed).

Definition2.17[13]:A subset A of (X, m_X) is said to be m_X – **Feebly regular clopen** if A = m_X –*f*.int (m_X – *f*.cl (m_X –*f*.int (A))). On the other hand, if A is m_X –*f*.reg.open and m_X –*f*.reg.closed.

Definition2.18[13]: Let A be a subset of (X, m_X). The m_X – **Feebly regular closure** of A (briefly m_X – *f*.reg.cl(A)) is the intersection of all m_X – Feebly regular closed sets containing A and the m_X –**Feebly regular interior** of A (briefly m_X –*f*.reg.int(A)) is the union of all m_X – Feebly regular open sets contained in A. The complement of m_X – Feebly regular open set is m_X – Feebly regular closed.

III m_X – Feebly regular open mappings with its complement

In this section we introduce m_X – Feebly regular open and m_X – Feebly regular closed mappings and some of its properties are discussed.

Definition 3.1: A function $f:(X,m_X)\to(Y,m_Y)$ is called the m_X – Feebly regular closed if the image of each m_X – Feebly regular closed in (X,m_X) is a m_X – Feebly regular closed in (Y,m_Y).

Definition 3.2: A function $f:(X,m_X)\to(Y,m_Y)$ is called m_X – Feebly regular open if the image of each m_X – open set in (X,m_X) is m_X – Feebly regular open set in (Y,m_Y).

Theorem 3.3: Every m_X – open mapping is m_X – Feebly regular open mapping.
Proof: Let $f:(X,m_X)\to(Y,m_Y)$ be a m_X – open mapping. Now we have to prove that f is m_X – Feebly regular open. Let H be any m_X – open subset of (X,m_X) . Since *f* is m_X – open mapping, *f*(H) is m_X – open in (Y,m_Y), *f*(H) is m_X – Feebly regular open. Hence *f* is m_X – feebly open mapping.

Theorem 3.4: Every m_X – closed mapping is m_X – Feebly regular closed mapping.
Proof: Let $f:(X,m_X)\to(Y,m_Y)$ be a m_X – closed mapping. Now we have to prove that *f* is m_X – Feebly regular closed mapping. Let H be any m_X – closed subset of (X,m_X). Since *f* is m_X – closed mapping, *f*(H) is m_X – closed in (Y,m_Y). *f*(H) is m_X – Feebly regular closed. Hence *f* is m_X – Feebly closed mapping.

Theorem 3.5: Let $f:(X,m_X)\to(Y,m_Y)$ be a m_X – Feebly regular closed mapping then the image of every m_X – closed subset of (X,m_X) is m_X – Feebly regular closed in (Y,m_Y).

Proof: Let H be any m_X – closed subset of (X,m_X) and *f*(H) is m_X – Feebly regular closed in (Y,m_Y) then *f*(H) is m_X – Feebly regular closed in (Y,m_Y).

Theorem 3.6: A mapping $f:(X,m_X)\to(Y,m_Y)$ is m_X – Feebly regular open if *f*(m_X –F.reg.int(H)) $\subseteq$ m_X – F.reg.int *f*(H) for every H $\subseteq$ (X,m_X).

Proof: Let H be any m_X – open set in (X,m_X). So that m_X –F.reg.int (H) = H, then *f*(m_X –F.reg.int(H)) $\subseteq$ m_X –F.reg.int *f* (H). Therefore *f*(H) $\subseteq$ m_X –F.reg.int *f* (H). But m_X –F.reg.int (*f* (H)) $\subseteq$ *f*(H) always. Hence m_X –F.reg.int (*f* (H)) = *f*(H). Therefore *f*(H) is m_X – open in (X,m_X), then *f* is m_X – open. By theorem 3.3, *f* is m_X – Feebly regular open.

Theorem 3.7: A mapping $f:(X,m_X)\to(Y,m_Y)$ is m_X – Feebly regular closed if m_X –F.reg.cl (f(H)) $\subset$ f(m_X –F.reg.cl (H)) for every H $\subset (X,m_X)$.

Proof: Let H be any m_X – closed set in (X,m_X). So that m_X –F.reg.cl(H) = H. By hypothesis m_X – F.reg.cl f(H) $\subset$ f(m_X –F.reg.cl(H))= f(H). Therefore, m_X –F.reg.cl (f(H)) $\subset$ f (H). But f (H) $\subset$ m_X – F.reg.cl f(H) always. Hence m_X –F.reg.cl f(H)= f(H), thus f(H) is m_X – closed, then f is m_X – closed map. By theorem 3.4, f is m_X – Feebly regular closed.

Theorem 3.8: Let $f:(X,m_X)\to(Y,m_Y)$ and $g:(Y,m_Y)\to(Z,m_Z)$ be a mappings then $g\circ f:(X,m_X)\to(Z,m_Z)$ is m_X– Feebly regular open if **(i)** f and g be the m_X–open mappings **(ii)** f is m_X– open and g is m_X – Feebly regular open mappings**.**

Proof: (i) Let H be any m_X–open subset of (X,m_X). Now we have to prove that $(g\circ f)$ (H) is m_X – Feebly regular open in (Z,m_Z). Since f is m_X–open, then f(H) is m_X –open in (Y,m_Y). Also we have g is m_X– open, then g(f(H)) is m_X–open in (Z,m_Z). Therefore ($g\circ f$) (H) is m_X– Feebly regular open in (Z,m_Z). Thus $g\circ f$ is m_X – Feebly regular open mapping.

(ii) By same method in part (i).

Remark 3.9:(i) Let $f:(X,m_X)\to(Y,m_Y)$ and $g:(Y,m_Y)\to(Z,m_Z)$ be the m_X –closed mappings then $g\circ f$: $(X,m_X)\to(Z,m_Z)$ is m_X – Feebly regular closed.**(ii)** Let $f:(X,m_X)\to(Y,m_Y)$ be m_X –closed and g: $(Y,m_Y)\to(Z,m_Z)$ be m_X– Feebly regular closed, then g∘f : $(X,m_X)\to(Z,m_Z)$ is m_X– Feebly regular closed.

4. m_X – Feebly regular continuous functions

Definition 4.1: A m_X –mapping $f:(X,m_X)\to(Y,m_Y)$ is said to be m_X – Feebly regular continuous if the soft inverse image by f of each m_X – open set H of (Y,m_Y) is m_X – Feebly regular open in (X,m_X).

Remark 4.2:(i) Let (X,m_X) be a m_X– topological space, A and B $\subset (X,m_X)$ if A$\subset$B, then f (m_X–F.reg.cl (A)) $\subset f$ (m_X –F.reg.cl (B)). (ii) Let (X,m_X) be a minimal topological space if A is m_X – Feebly regular open if and if only A^c is m_X– Feebly regular closed. From our definition of m_X – Feebly regular open and m_X**–** Feebly regular closed sets, we obtain them.

Theorem 4.3: If $f:(X,m_X)\to(Y,m_Y)$ is m_X – Feebly regular continuous if and if only the inverse image of every m_X–closed subset of (Y,m_Y) is m_X – Feebly regular closed in (X,m_X).

Proof: We have f is m_X– Feebly regular continuous. Let H is m_X–closed in (Y,m_Y), H^c is m_X–open in minimal structure, $f^{-1}(H)^c=(f^{-1}(H))^c$ is m_X – Feebly regular open in (X,m_X), then by remark 4.2(ii) f^{-1}(H) is m_X– Feebly regular closed. H is a m_X –open set in (Y,m_Y), H^c is m_X –closed, then by hypothesis $f^{-1}(H)^c$ is m_X – Feebly regular closed in (X,m_X), then by remark 4.5(ii) f^{-1}(H) is m_X – Feebly regular open set in (Y,m_Y). Thus f is m_X – Feebly regular continuous.

Theorem 4.4: Every m_X –continuous mapping is m_X – Feebly regular continuous mapping.

Proof: Let $f:(X,m_X)\to(Y,m_Y)$ is m_X – continuous mapping. Now we have to prove that f is m_X – Feebly regular continuous. Let H be any m_X – open subset of (Y,m_Y). Since f is m_X –continuous then f^{-1}(H) is m_X–open in (X,m_X). Therefore f^{-1}(H) is m_X– Feebly regular open. Hence f is m_X– Feebly regular continuous mapping.

Theorem 4.5: Let (X, m_X) be a m_X –topological space, A $\subset$ (X, m_X) , then A is m_X – Feebly regular open if and if only f m_X –F.reg.int (A) = A.

Proof: We have A is m_X – Feebly regular open set in (X, m_X). It is clear *f*(m_X –F.reg.int (A)) $\subset$ A-→(1). Since A is m_X – Feebly regular open set and *f* (m_X –F.reg.int (A)) is largest m_X – Feebly regular open set. Then A $\subset$ *f*(m_X –F.reg.int (A))----→(2). From (1) and (2) we obtain *f*(m_X –F.reg.int (A)) =A. Conversely let *f* (m_X –F.reg.int (A)) =A. Since *f*(m_X –F.reg.int (A)) is m_X – Feebly regular open set, then A is m_X – Feebly regular open set.

Corollary 4.6: Let (X, m_X) be a m_X – topological space, A $\subset$ (X, m_X) then A is m_X – Feebly regular closed if and if only *f*(m_X –F.reg.cl (A)) = A.

Theorem 4.7: If $f:(X, m_X) \to (Y, m_Y)$ is m_X – Feebly regular continuous if and if only *f*(*f*(m_X –F.reg.cl (A))) $\subset$*f*(A^c) for every A $\subset (X, m_X)$.

Proof: We have *f* is m_X – Feebly regular continuous. Since *f*(A^c) is soft-closed in (Y, m_Y), then by theorem 4.3, f^{-1}(*f*(A^c)) is m_X – Feebly regular closed in (X, m_X). By corollary 4.6, *f*(m_X –F.reg.cl (f^{-1}(*f*(A^c))))= f^{-1}(*f*(A^c))--→(1). Now *f*(A) $\subset$ *f*(A^c) $\Rightarrow$ A $\subset$ f^{-1}(*f*(A)) then A $\subset$ f^{-1}(*f*(A^c)), thus by remark 4.2(i), *f*(m_X –F.reg.cl (A)) $\subset$ *f*(m_X –F.reg.cl (f^{-1}(*f*(A^c)))) according to (1), we get *f*(m_X –F.reg.cl (A)) $\subset$ f^{-1}(*f*(A^c)), then *f*(*f*(m_X –F.reg.cl(A))) $\subset$(*f*(A))^c . Conversely, let *f*(*f*(m_X –F.reg.cl(A))) $\subset$(*f*(A))^c for every A $\subset (X, m_X)$. Let H is m_X –closed set in (Y, m_Y). Then H^c = H, let f^{-1} (H) be any m_X –subset of (X, m_X), then by hypothesis *f*(m_X –F.reg.cl (f^{-1}(A))) $\subset$ *f*((f^{-1}(H))^c) =H^c= H. Thus *f*(m_X –F.reg.cl (f^{-1}(H))) $\subset$ f^{-1}(H) but f^{-1}(H)$\subset$ *f*(m_X – F.reg.cl (f^{-1}(H))) always thus f^{-1}(H)= *f*(m_X –F.reg.cl (f^{-1}(H))). Therefore by corollary 4.6, f^{-1}(H) is m_X – Feebly regular closed in (X, m_X), hence by theorem 4.6, f is m_X – Feebly regular continuous.

Theorem 4.8: If $f:(X, m_X) \to (Y, m_Y)$ is m_X – Feebly regular continuous if and if only *f*(m_X –F.reg.cl (f^{-1}(B))) $\subset$$f^{-1}$($B^c$) for every B $\subset (Y, m_Y)$.

Proof: We have *f* is m_X – Feebly regular continuous. Since B^c is m_X –closed in (Y, m_Y). Then by theorem 4.3, f^{-1}(B^c) is m_X – Feebly regular closed in (X, m_X) and by corollary 4.6, *f*(m_X –F.reg.cl (f^{-1}(B^c))) = f^{-1}(B^c) ---→(1). Now B $\subset$$B^c$$\Rightarrow$ f^{-1}(B) $\subset$ f^{-1}(B^c) then by remark 4.2(i), *f*(m_X –F.reg.cl (f^{-1}(B))) $\subset$ *f*(m_X –F.reg.cl (f^{-1}(B^c))), according to (1) we get, *f*(m_X –F.reg.cl (f^{-1}(B))) $\subset$ f^{-1}(B^c). Conversely, let *f*(m_X –F.reg.cl (f^{-1}(B))) $\subset$ f^{-1}(B^c) for every B $\subset (Y, m_Y)$. Let H be any m_X –closed in (Y, m_Y). Then H^c = H by hypothesis *f*(m_X – F.reg.cl (f^{-1}(H))) $\subset$ f^{-1}(H^c) = f^{-1}(H). Thus *f*(m_X –F.reg.cl (f^{-1}(H))) $\subset$ f^{-1}(H) , but f^{-1}(H) $\subset$ *f*(m_X –F.reg.cl (f^{-1}(H))), therefore *f*(m_X –F.reg.cl (f^{-1}(H))) = f^{-1}(H). Then by corollary 4.6, f^{-1}(H) is m_X – Feebly regular closedin (X, m_X), hence by theorem 4.3, f is m_X – Feebly regular continuous.

Theorem 4.9: Let $f:(X, m_X) \to (Y, m_Y)$ be a m_X –mapping if *f*(m_X –F.reg.cl (H)) $\subset$ *f*(m_X –F.reg.cl *f*(H)) for every H $\subset (X, m_X)$ then *f* is m_X – Feebly regular continuous.

Proof: Let H be any m_X –closed set in (Y, m_Y), then by remark 4.2(ii), let H is m_X – Feebly regular closed so that by corollary , *f*(m_X –F.reg.cl (H)) = H, f^{-1}(H) is a m_X –subset of (X, m_X) so that by hypothesis *f*(m_X –F.reg.cl (f^{-1}(H))) $\subset$ *f*(m_X –F.reg.cl(*f*(f^{-1}(H)))) = *f*(m_X –F.reg.cl(H)) = H. Therefore *f*(m_X –F.reg.cl (f^{-1}(H))) $\subset$ f^{-1}(H) always. Hence *f*(m_X –F.reg.cl (f^{-1}(H))) =f^{-1}(H) then by corollary 4.6, f^{-1}(H) is m_X – Feebly regular closed in (X, m_X). Therefore by theorem 4.3, f is m_X – Feebly regular continuous.

Theorem 4.10: Let $f:(X,m_X)\rightarrow(Y,m_Y)$ be a m_X –mapping if *f*(m_X –F.reg.cl (f^{-1}(H))) $\subset f^{-1}$(*f*(m_X –F.reg.cl(H))) for every H $\subset(Y,m_Y)$ then *f* is m_X – Feebly regular continuous.

Proof: Let H be any m_X –closed set in (X,m_X) then by result 3.5, we have H is a m_X – Feebly regular closed set and so by corollary 4.6, *f*(m_X –F.reg.cl(H)) =H. By hypothesis *f*(m_X –F.reg.cl (f^{-1}(H))) $\subset f^{-1}$(*f*(m_X –F.reg.cl(H)))= f^{-1}(H). Therefore m_X –F.reg.cl (f^{-1}(H)) $\subset f^{-1}$(H). But f^{-1}(H) $\subset$ *f*(m_X –F.reg.cl (f^{-1}(H))) always. Hence *f*(m_X –F.reg.cl (f^{-1}(H)))= f^{-1}(H) then by corollary 4.6, f^{-1}(H) is m_X – Feebly regular closed in (X,m_X). Therefore by theorem 4.3, f is m_X – Feebly regular continuous.

III CONCLUSIONS

In further work, at these mappings based on these sets with its related points will be generalized and also extended.

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