On the Topological Seismography through Ricci Flow Approach

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ABSTRACT

In this paper, the earthquake is studied topologically. The parameters of seismography are defined over the three dimensional Riemannian manifolds. The Ricci flow theory is applied to set the topological seismograph. The accuracy of seismograph is increase with this approach by the three dimensional transformation. The three dimensional Riemannian manifolds interact with the earthquake domain. The solving the manifolds problem is equivalent to the solving the earthquake problem. There are finite parameters in seismography but these parameters are defined by algebraic approach only. This paper defines the seismography algebraically and geometrically both for achieving the accurate result. This is possible by metric measurement of earthquake. The tensor application minimizes the error in the measurement.

Key Words: Earthquake, Seismography, Riemannian manifolds, Ricci flow, Metric, Tensor.

1. **Introduction:** The movement of earth is measured by seismograph. The up-down motions of the earth are transformed in the paper as a graph by the device. This device is called seismometer and this mechanism is called seismography. In 2006, Alberti [2] studied the dataset of earthquake over spatial structures. There exists the fault defined by the quantifying similarity method. In 1997, Amelung [3] presented the report on small earthquakes based on large scale tectonic deformation. Backus et al. [5-7] formulated the earthquake and defined its trajectory over the seismic sources. Kagan [17-27] is credited to develop the topological seismography which becomes the inspiration of this study. The tensor based study of earthquake, predict the earthquake over the focal method, the probabilistic study of earthquake etc. are the noteworthy research on seismography. In 2005, Anderson [4] presented the fracture mechanics. Later its mass applications appeared. The plate tectonics and plate reconstruction have the cumulative domain occurred in [8-10]. The earthquake database is distributed over the deformation of plate database and its mapping is studied through the manifolds conditions. Chen [11] defined the earthquake through the finite moment tensor domain. Clinton et al. [12] surveyed the effectiveness of seismography under the conditions of tensor propositions. In 1989, Frohlich [13] analyzed the non-double couple earthquake. {14-16] are the geometric study of seismography. Kubo et al. [33] streamed the seismic moment tensor in Japan region. In 2004, Kuffner [34] transformed the seismic domain into the metrics for approximating the computation. [30-3, 35] are the noteworthy research towards the mathematical study of seismography. Next, topological seismography is proposed.

2. **Topological Seismography:** In this section seismography is studied topologically. The seismography and Ricci flow correspondence is defined for analyzing seismography through manifolds. It lies with the topology and geometry of n-manifolds. The Ricci flow associates with n-dimensional Riemannian

manifold. Let M^n be an n- dimensional Riemannian manifold with g_0 metric. If, time ($t \geq 0$), g(t) is the time dependent family ,then the Ricci flow (M^n , g_o) over g(0)= g_0 is defined by the following Ricci flow equation

$$\frac{\partial}{\partial t} g(t) = - 2\, Ric^{g_t} \text{……………………………..……………………………..(1)}$$

Where Ric $^{g(t)}$ is the Ricci curvature tensor, t∈ [o,T] .

Let S^3 be a sphere of dimension -3 This sphere is connected with a stainless steel wire and hooked with the top surface of a glass cube for free oscillation as follows

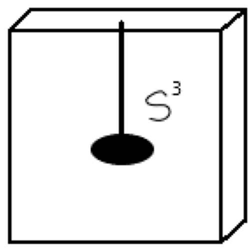


Fig 1 S^3 - Cube

There is a finite time T>0, at this point the oscillation (flow) is singular and shrinks to a point mathematically, the metric approaches to 0 and the curvature is unbounded everywhere. In seismolographical application there are two cases exists, first the S^3 – is stable state and second, S^3 is an unstable state. The metric approaches to 0 for the first case but the second case, the metric value will be non-zero. The following figure corresponds with this metric cases.

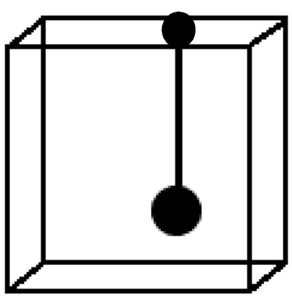


Fig. 2 Stable case(d=0)

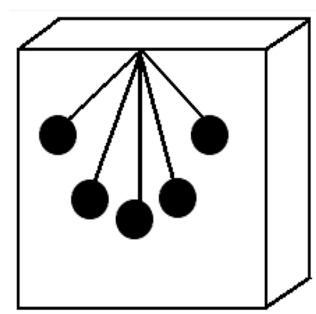


Fig. 3 Unstable case (d$\neq$ 0)

The measurement of curvature ,lies with the two factors , recalling $\rho(t)$ and the normalized $\rho(t)g(t). The$ convergence is defined by t→ T .

The Ricci flow approach defines the freely moving S^3 topologically. This freely moving characteristics corresponds with the earthquake. Definitely, seismography measures the earthquake efficiently but its n-dimensional measurements becomes feasible and optimal by the proposed topological Ricci flow method. Thus, this method applies to measure the earthquake efficiently with n- dimensional Riemannian manifold as the substitution of general seismography.

The primary phase of this approach presents as follows:

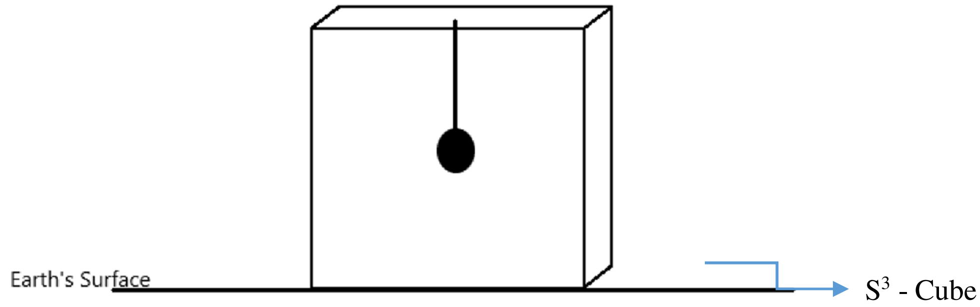


Fig. 4 Without earthquake

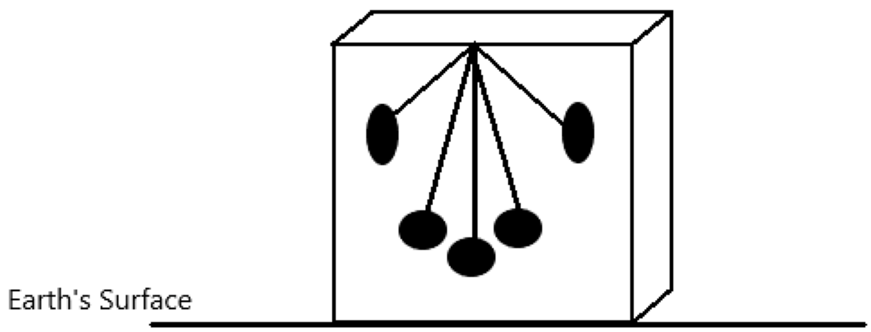


Fig .5 With Earthquake

If the S^3 – moving freely during the earthquake then, its path moves as follows

Fig 6 Path – Curve during earth quake of S^3

The natural question exists, what will be the equation of this path-curve? This study focuses on this objective by Ricci flow. This is same as the graph of the seismograph which is defined by the seismographic parameters. In this path –curve the equation interacts with the (M^n, g_0). This equation carries the initial value, middle value and boundary value conditions. Manifold theory transforms the case in any dimension. The same case also transforms into the 2D-surface. The result of S^3- freely moving case (3D) and the result of S^3 –freely moving (2D) can be compared for minimizing the error. The S^3 –freely moving in 2 D –surface in the same glass cube or separate glass cube is illustrated as follow

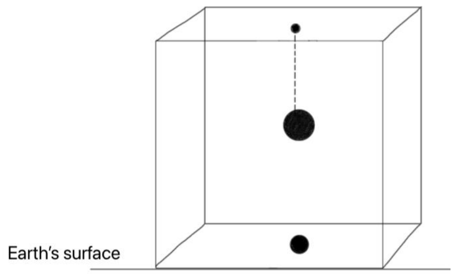


Fig.7 S^3 -2D -3D

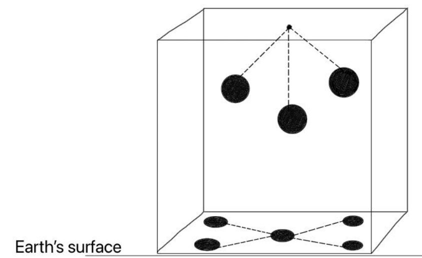


Fig .8 S^3 -2D -3D

Its generalization is presented in following figure by the separation of S^3 -2D and S^3 -3D simultaneously

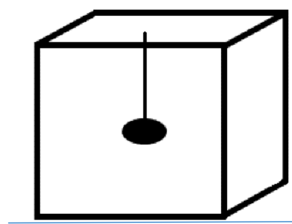


Fig.9 S^3 -3D

(Stable)

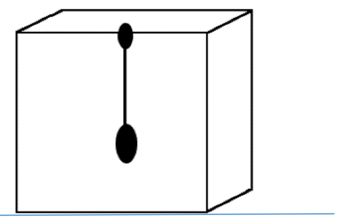


Fig.10 S^3 -2D

(Stable)

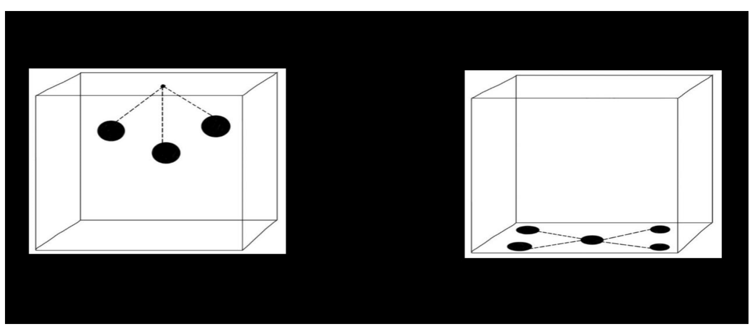


Fig. 11 S^3 -3D **Fig. 12 S^3 -2D**

(Unstable)

The key observation will be analyzed on the homogeneity of curve which can be understood through the following figure.

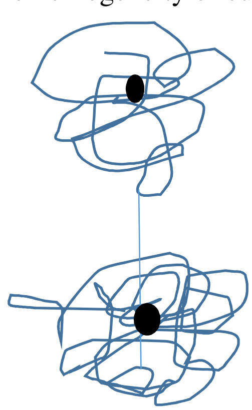


Fig. 13 S^3 -3D -2D Isomorphism

The isomorphic study comprises with the minimization of error and maximization of the accuracy of measuring and predicting the earthquake.

First, S^3-2D equation to be derived. The structure of the path of S^3 during the earthquake is studied by Ricci flow equation. The S^3 is kept in the center of a closed glass cylinder (fig-14 a & b)

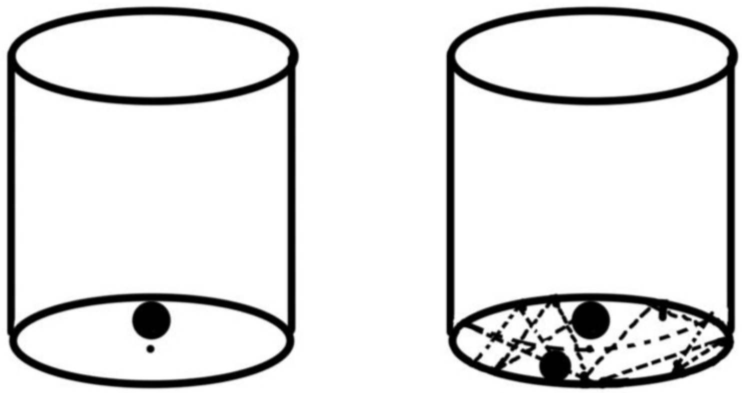
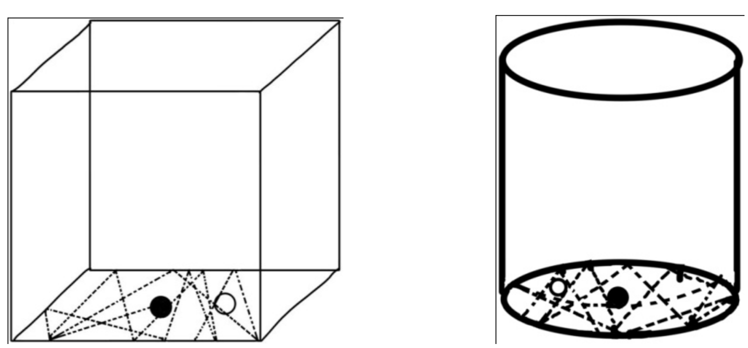


Fig.14 (a)S^3- Position (Stable) **Fig.14 (b) S^3 –Position(unstable)**

An interesting result is noticed here, i. e. previously the closed glass cube was taken, but in fig14(a &b), the experiment is performing in closed cylindrical glass. In Riemannian geometry, the structure (cylinder or cube) is isomorphic by manifold theory. It means the cube and cylinder can be transformed into each other. The objective of talking these two distinct structures for measuring and predicting the earthquake accurately. Definitely the surface area or outer structures of both the shapes are distinct, but with manifold or topological context, both the structures are transformable into each other. During earthquake, S^3-generates the curves in both the settings, cube and cylinder as follows

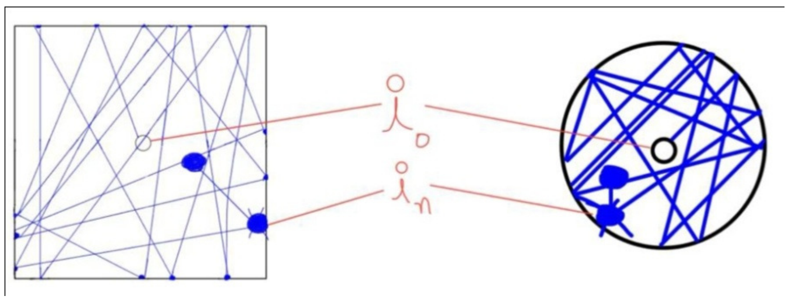


(a) S^3 – Cube **(b) S^3 - Cylinder**

Fig. 15 S^3- Curves

The objective of this paper is to study both the curves for defining the earthquake topologically. The prediction of earthquake is studied also over the curves of n- earthquakes. The comparison of the curves of 2 or more than 2 earthquakes sets the foundation to predict the earthquake also. In each earthquake, the curves of both the settings will be studied by Ricci flow. The equation of each curve and its similarities and differences set the decision on the measuring and predicting the earthquake.

In any earthquake, time t – second, C_1, C_2 are the curves of cube and cylinder respectively. i_0, i_n are the initial and final positions of S^3 respectively in cube and cylinder both.



(a) **C_1** **(b) C_2**

Fig. 16 S3- Curves

First an equation of C_1 to be derived

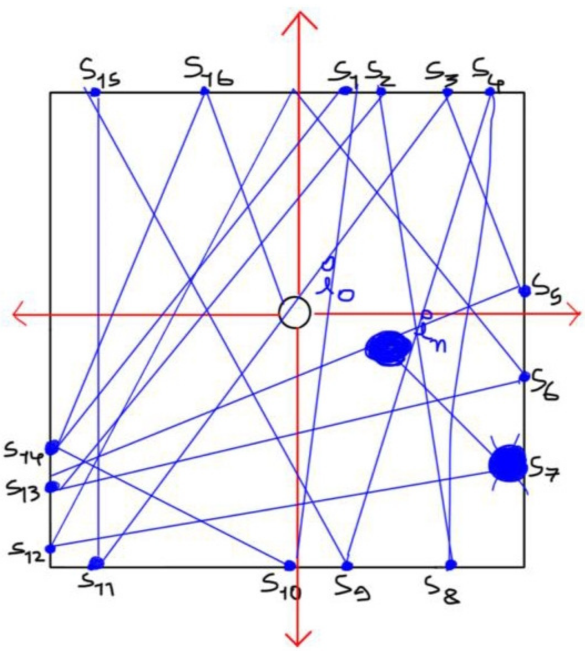


Fig. 17 C_1

S_1, S_2, ………., S_{16} are the points of contact in edges $e_{n=7.}$ At t_0, s^3 is in i_0, at t_1, s^3 is in s_1, similarly at t_n, s^3 is in S_n or S_{16}(for fig .17). Thus, there are two sets, S = {S_1, S_2,, Sn} and T = {t_1, t_2,……,t_n} defined correspondingly. The above sets are also represented as the time dependent set,

S = {$S_1(t_1)$, $S_2(t_2)$, ……., $S_n(t_n)$}. The inter relationship among its elements is the primary objective. Ricci flow assignment over each time t, i. e. t_1, t_2, …. tn. Let the rectangular domain be a smooth manifold M, g be a Riemannian metric defined for measuring the curvature. Ric^g be a Ricci tensor , the tangent to the curve is defined by the point $p \in M$, g_p is a positive definite inner product on the T_pM, T_pM be a tangent space, then there exists the partial derivative $\frac{\partial}{\partial t}g_t$, for every Riemannian metrics g_t . There exists an open interval (a, b) in the rectangular domain. Then, the Ricci flow equation

$\frac{\partial}{\partial t}g_t$= -2 Ric^{g_t} its generalization for set t = {t_1, t_2, …..,t_n} is defined by

$$\frac{\partial}{\partial t_1}g_{t_1}= -2Ric^{g_{t_1}}$$

$$\frac{\partial}{\partial t_2}g_{t_2} = -2Ric^{g_{t_2}}$$

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$$\frac{\partial}{\partial tn}g_{t_n}= -2\ Ric^{g_{t_n}}$$

Suppose , g_t be a solution of the Ricci flow with t ∈ (0,T) over the bounded curvature. Then, each g_t has the curvature operator, this curvature operator will be non-negative. For any curve in the rectangular domain Y or Y(t): [t, t_{n+1}] → M with [t_n , t_{n+1}]∈(0, T) , then

$$\frac{d}{dt}\,(\mathrm{Rec}^{ut}(y(t))+\frac{Ric^{g_t}}{t}(y(t)) +\frac{1}{2}Ric^{g_t} .$$

Its generalization over the time set t = t_1 ,t_2 ,………, t_n ,

$$\frac{d}{dt_1}(Ric^{g_{t_1}}\ (y(t)) + \frac{Ric^{g_{t_1}}}{t_1}\,(y(t_1))+\frac{1}{2}Ric^{g_{t_1}}$$

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$$\frac{d}{dt_n}(Ric^{g_{t_n}}(y(t))+\frac{Ric^{g_{t_n}}}{t_n}\,(y(t_n)) +\frac{1}{2}Ric^{g_{t_n}}.$$

Its solution comprises with the following linear algebraic presentation,

$$\begin{bmatrix} \frac{\partial}{\partial t}(a_{11}, b_{11})g_t\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(a_{mn}, b_{mn})g_t \end{bmatrix} = -2Ric^{g_t}$$

Where , (a_{ij}, b_{ij}) is the corresponding co-ordinate of s_n.

Its generalization

$$\begin{bmatrix} \frac{\partial}{\partial t}(a_{11,}b_{11})g_{t_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \\ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(a_{mn}, b_{mn})g_{t_1} \end{bmatrix} = -2Ric^{g_{t_1}}$$

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$$\begin{bmatrix} \frac{\partial}{\partial t_n}(a_{11}, b_{11})g_{t_n}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \\ : \\ \\ :\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(a_{mn},b_{mn})g_{t_n} \end{bmatrix} = -2Ric^{g_{tn}}$$

The linear algebraic notation of curvature operator as follows:

$$\begin{bmatrix} \frac{d}{dt}(a_{11}, b_{11})Ric^{g_t}(y(t))\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \\ \ldots\ldots\ldots\ldots\ldots\ldots(a_{mn}, b_{mn})Ric^{g_t}(y(t)) \end{bmatrix} = - \frac{Ric^{g_t}}{t}(y(t)) - \frac{1}{2}Ric^{g_t}$$

$$\begin{bmatrix} \frac{d}{dt_1}(a_{11}, b_{11})Ric^{g_{t_1}}(y(t_1))\ldots\ldots\ldots\ldots\ldots\ldots \\ \\ \ldots\ldots\ldots\ldots\ldots\ldots(a_{mn}, b_{mn})Ric^{g_{t_1}}(y(t_1)) \end{bmatrix} =- \frac{Ric^{g_{t_1}}}{t_1}(y(t_1)) - \frac{1}{2}Ric^{g_{t_1}}$$

$$\begin{bmatrix} \frac{d}{dt_n}(a_{11}, b_{11})Ric^{g_{t_n}}(y(t_n))\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \\ \ldots\ldots\ldots\ldots\ldots\ldots(a_{mn}, b_{mn})Ric^{g_{t_n}}(y(t_n)) \end{bmatrix} = - \frac{Ric^{g_{t_n}}}{t_n}(y(t_n)) - \frac{1}{2}Ric^{g_{t_n}}$$

The Riemannian manifolds is defined on the set of surface points

S = $\{s_1(t)$, s_2 (t) , …………., $s_n(t)$ }. Since the curve and velocity vectors are independent, therefore the tangent vectors is defined by

$\frac{d}{dt}$\ J=O f 0 y(t) , y(0) = s_m , m = 0 , 1 , 2 ……., n.

Then, a non- homogeneous linear constant coefficient system of equations of the form

$y'_1 = a_{11}y_1 + a_{12}y_n + \ldots\ldots\ldots\ldots + a_{1n}y_n + n_1$(t)

.

.

$y'_n = a_{m1}y_1 + a_{m2}y_2 + \ldots\ldots\ldots\ldots + a_{mn}y_n + n_n$(t)

where , y = $\begin{matrix} y_1 \\ y_2 \\ y_n \end{matrix}$, A = $\begin{bmatrix} a_{11} \ldots\ldots\ldots\ldots\ldots\ldots \\ \\ \ldots\ldots\ldots\ldots\ldots a_{nn} \end{bmatrix}$

h(t) = (h_1(t) , …………….. , h_n(t)) ;

h_1(t) = - $2Ric^{g_t}(y_1(t))$,

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h_n(t) = - $2Ric^{g_t}(y_n(t))$.

$y_1(t)$:$[t_0, t_1]$ → M ; $[t_0, t_1]$ ∈(0, T) ,

.

$y_n(t)$:$[t_{n-1}, t_n]$ → M ;$t[_{n-1}, t_n]$ ∈(0, T) .

Similarly B = $\begin{bmatrix} b_{11} \ldots\ldots\ldots\ldots\ldots\ldots \\ \\ \ldots\ldots\ldots\ldots\ldots b_{nn} \end{bmatrix}$

The general solution of the system is

Y(t) = $A_1 Y^*_1 + B_1 Y^*_2 = A_1 e^{\lambda_1 t} x^{(1)} + B_1 e^{\lambda_2 t} x^{(2)}$, where , $x^{(1)}$= $(x_{11}, x_{12})^T$, $x^{(2)}$= $(x_{21}, x_{22})^T$,

Y_1 (t) = = $A_1 e^{\lambda_1 t} x_{11} + B_1 e^{\lambda_2 t} x_{21}$, Y_2(t)= $A_1 e^{\lambda_1 t} x_{12} + B_1 e^{\lambda_2 t} x_{22}$;

$Y^*_1 = e^{\lambda_1 t} x^{(1)}$, $Y^*_2 = e^{\lambda_2 t} x^{(2)}$,

For non –homogeneous system, by method of undetermined coefficients or the method of diagonalization to find the particular integral. Thus the earthquake transforms into the equation of manifolds through Riemannian metric and simultaneous system of linear differential equation over the Ricci flow conditions for curvature operator y(t). The problems of earthquake measuring and predicting interact with the problem of solving system of linear differential equation. If this is solved, then earthquake is studied, i.e. measured and predicted. Next, the flow chart and algorithm of this flow chart.

3. **The Algorithm and Flow Chart of Topological Seismography:** The algorithm of the topological seismograph is presented as follows:

The algorithm for measuring and predicting the earthquake based on flow equation over Riemannian metric as follows:

1. Start.
2. Input set of surface points

   S = {$s_1(t_1)$, s_2 (t_2), ………, s_n (t_n)}.

3. Ricci flow metric: Ric^{g_t}
   $Ric^{g_{t_1}}, Ric^{g_{t_2}}, \ldots \ldots \ldots \ldots . . Ric^{g_{t_n}}$.
4. Define the curvature operator y(t): $[t_n, t_{n+1}] \rightarrow M$;$[t_n, t_{n+1}] \in (0, T)$.
5. Does y(t) exists;
   I f no then go to (7)
   If yes, then go to next.
6. System of linear differential equation.
7. General solution; y(t).
8. End.

The corresponding flow chart of this algorithm as follows:

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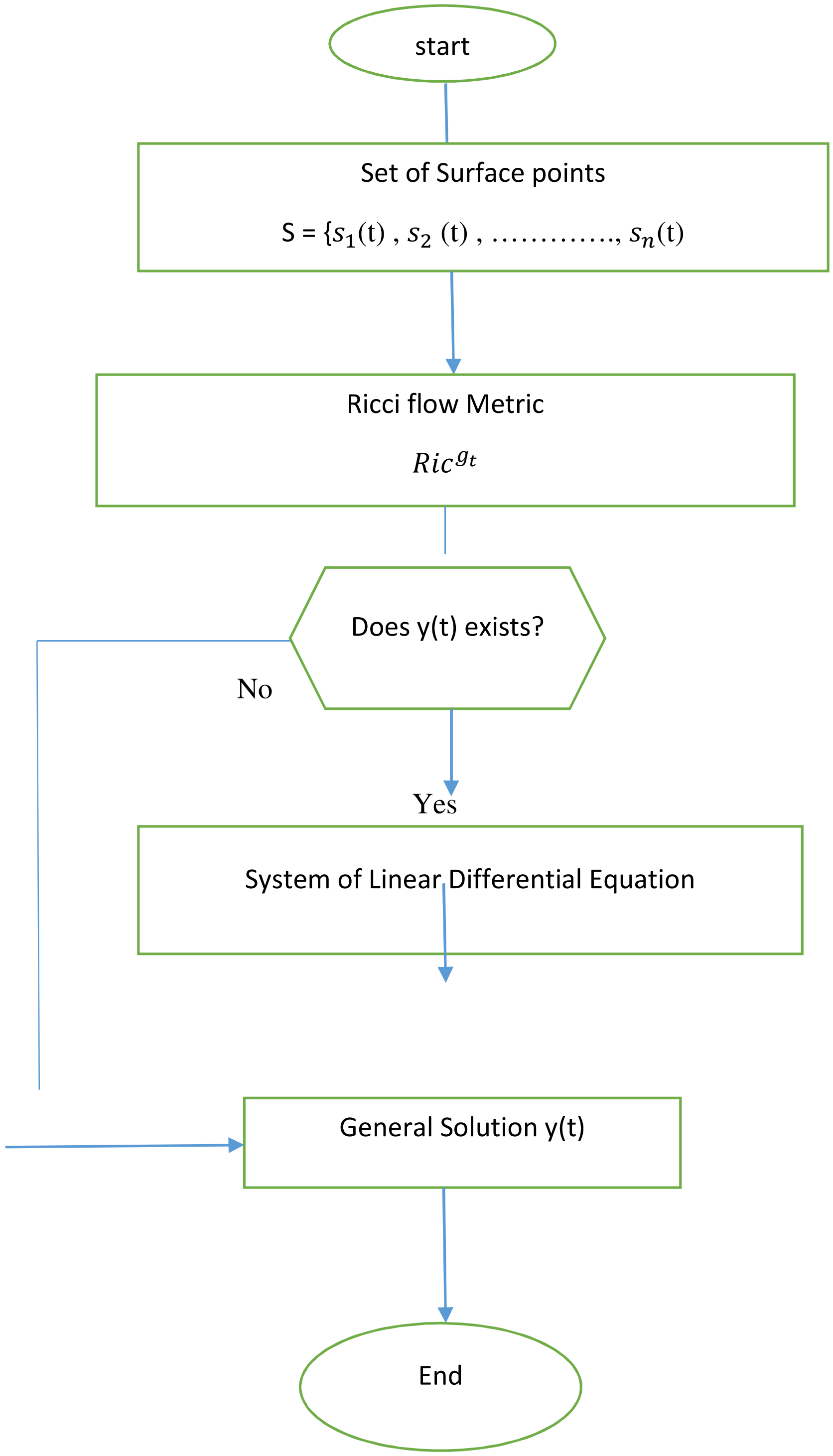

4. Conclusion:

Ricci flow equation through manifolds over the Riemannian metric transforms the earthquake curve formed in a bounded rectangle into system of linear differential equation. The solution of this system corresponds with the solution of the earthquake problem. This method applied to the measure and predict the earthquake efficiently. The generalization of this method is also discussed in this paper. The generalization carries the bounded cylinder, curves in 2D and 3D spaces and bounded rectangle 3D space. Each method can be studied by Riemannian metric. Ricci flow equation sets the correspondence between points of the curve and its manifolds. 2D – Manifolds are transformed feasibly and optimally by 2D – Riemannian metric but 3D – Riemannian metric has also the trivial solution for certain 3D- Manifolds but major 3D – Manifolds have the non-trivial solution. The flow chart and algorithm of the proposed method illustrate the idea.

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