NEIGHBORHOOD-PRIME LABELING OF CORONA PRODUCT OF TWO GRAPHS

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Abstract: We discuss the neighborhood-prime labeling of the corona product of an arbitrary graph G with graphs H of specific families. In particular, we show that the corona product of an arbitrary graph G with path, cycle, union of two paths, union of two cycles and a n-polygonal book graph is neighborhood-prime. We also show that the corona product of G with any graph whose total domination number is at most 7 is neighborhood-prime. As a consequence we deduce that the corona product of G with wheel graph, complete graph and certain types of complete bipartite graphs is neighborhood-prime.
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1 Introduction

By a graph, we shall always mean a simple, finite and an undirected graph. We denote the vertex set of a graph G by $V(G)$ and its edge set by $E(G)$. The cardinalities of these two sets will be denoted by $|V(G)|$ and $|E(G)|$ respectively. For an arbitrary vertex v of a graph G, the notation $N_G(v)$ is used for the set of vertices in G which are adjacent to v. This set is known as the neighborhood of v and when the context of the graph is clear, it is simply denoted by $N(v)$.

Definition 1.1. *A bijective function* $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ *is said to be a neighborhood-prime labeling of a graph G if* $\gcd(f(N(v))) = \gcd\{f(u) : u \in N(v)\} = 1$*, for every vertex* $v \in V(G)$ *whose degree is at least* 2*. A graph that admits neighborhood-prime labeling is called a neighborhood-prime graph.*

The notion of a neighborhood-prime graph was first introduced by Patel and Shrimali [5] as a natural variant of prime labeling where it is required that $f(u)$ and $f(v)$ are relatively prime whenever u and v are adjacent vertices. In their initial research ([5],[6],[7]) they have shown that path, helm, closed helm, flower, union of two wheels, union of a finite number of paths, cartesian and tensor product of two paths are neighborhood-prime graphs. Moreover, they showed that a cycle C_n is neighborhood-prime iff $n \not\equiv 2 \pmod 4$ and characterized neighborhood-prime graphs amongst the class of union of two cycle graphs.

Neighborhood-prime labeling in the context of trees have been studied by Cloys and Fox [2]. They have shown that firecrackers, caterpillars, spiders, trees without degree 2 vertices are neighborhood-prime and later made a conjecture that all trees are neighborhood-prime. This makes the study of neighborhood-prime labeling more interesting because the same conjecture for trees in the context of prime labeling has remain unsolved for almost four decades. Asplund et al. [1] have shown that the generalized Petersen graphs are neighborhood-prime. Ghodasara and Patel [4] have proved that one point union of k copies of cycle, the barycentric subdivision of wheels and gears, the middle and total graph of crowns, the square of crowns, tadpoles, cycles and umbrellas are neighborhood-prime graphs. A brief summary of results related to prime labeling and neighborhood-prime labeling is available in the dynamic survey of graph labeling by J. Gallian [3]. In this paper, we focus on neighborhood-prime labeling of the corona product of two graphs which is defined as follows.

Definition 1.2. *The corona product of graphs G and H is the graph $G \odot H$ obtained by taking one copy of G (called the center graph) and $|V(G)|$ number of copies of H (called the outer graph) and making the i^{th} vertex of G adjacent to every vertex of the i^{th} copy of H, where $1 \le i \le |V(G)|$.*

For instance, a crown graph (obtained by adding pendent edges at every vertex of a cycle) can be considered as a corona product of a cycle with K_1. In the next section we prove that the corona product of an arbitrary graph G with path, cycle, union of two paths, union of two cycles and an n-polygonal book graph is a neighborhood-prime graph. We also show that the corona product of G with any graph whose total domination number is at most 7 is neighborhood-prime. As a result we deduce that the corona product of an arbitrary graph G with a wheel graph, a complete graph and complete bipartite graphs $K_{m,n}$ (with $m \le 7$) is neighborhood-prime. Finally we prove that if G is an arbitrary graph and H is either a comb graph $P_n \odot K_1$ or a crown graph $C_n \odot K_1$, then $G \odot H$ is neighborhood-prime under certain assumptions.

2 Main Results

Throughout the section, while discussing the neighborhood-prime labeling of the corona product $G \odot H$, we take $V(G) = \{u_1, u_2, \ldots, u_p\}$, where $p = |V(G)|$. Further, W_i shall denote the subgraph of $G \odot H$ which is induced by the vertex u_i of G along with all the vertices of the i^{th} copy of H say, H_i.

Definition 2.1. *If k is any positive integer then a bijective function $f : V(G) \to \{k, k+1, \ldots, k+|V(G)|-1\}$ is said to be a k-neighborhood-prime labeling of a graph G if $\gcd(f(N(v))) = \gcd\{f(u) : u \in N(v)\} = 1$, for every vertex $v \in V(G)$ whose degree is at least 2. A graph that admits k-neighborhood-prime labeling is called a k-neighborhood-prime graph.*

Thus a 1-neighborhood-prime labeling is nothing but neighborhood-prime labeling. The following lemma shows that neighborhood-prime labeling of the corona product $G \odot H$ can be obtained through k-neighborhood-prime labelings of the subgraphs W_i.

Lemma 2.1. *The corona product $G \odot H$ is a neighborhood-prime graph if each W_i is a $(k_i + 1)$-neighborhood-prime graph where $k_i = (|V(H_i)| + 1)(i - 1)$ for $1 \le i \le |V(G)|$.*

Proof. As each W_i is a $(k_i + 1)$-neighborhood-prime graph, there exists a bijective function $f_i : V(W_i) \to \{k_i + 1, k_i + 2, \ldots, k_i + |V(W_i)|\}$ such that $\gcd\{f_i(u) : u \in N_{W_i}(v)\} = 1$, for every vertex $v \in W_i$ whose degree is at least 2. Now define a mapping f on the vertex set of $G \odot H$ by defining $f(u)$ as $f_i(u)$, if u is a vertex of W_i. Then it is easy to verify that f is a neighborhood-prime labeling on $G \odot H$. □

Theorem 2.1. *If G is an arbitrary graph and H is a path P_n, then $G \odot H$ is neighborhood-prime graph for all n.*

Proof. We assume that $n \ge 4$ because the other cases follow with minor modifications which are left to the reader. In view of Lemma 2.1 it is enough to show that each W_i is a $(k_i + 1)$-neighborhood-prime graph. Here we consider the vertex set and the edge set of H_i as $V(H_i) = \{v^i_j : j = 1, 2, \ldots, n\}$ and $E(H_i) = \{v^i_j v^i_{j+1} : j = 1, 2, \ldots, n - 1\}$ respectively whereas the ith vertex of G is denoted by u_i.

Define $f : V(W_i) \to \{k_i + 1, k_i + 2, \ldots, k_i + |V(W_i)|\}$ as per the following two cases:

Case 1: n is odd.

$$\begin{aligned}
f(u_i) &= k_i + 2, \\
f(v^i_2) &= k_i + 1, \\
f(v^i_{n-1}) &= k_i + 3, \\
f(v^i_{2j-1}) &= k_i + 3 + j; && j = 1, 2, \ldots, \frac{n+1}{2}, \\
f(v^i_{2j}) &= k_i + \frac{n+7}{2} + (j - 1); && j = 2, 3, \ldots, \frac{n-3}{2}.
\end{aligned}$$

Case 2: n is even.

$$\begin{aligned}
f(u_i) &= k_i + 2, \\
f(v^i_2) &= k_i + 1, \\
f(v^i_{n-1}) &= k_i + 3, \\
f(v^i_{2j-1}) &= k_i + 3 + j; && j = 1, 2, \ldots, \frac{n-2}{2}, \\
f(v_{2j}) &= k_i + \frac{n+4}{2} + (j - 1); && j = 2, 3, \ldots, \frac{n}{2}.
\end{aligned}$$

Observe that $\gcd(f(N(v^i_1))) = \gcd\{f(u_i), f(v^i_2)\} = \gcd\{k_i + 2, k_i + 1\} = 1$ and $\gcd(f(N(v^i_n))) = \gcd\{f(u_i), f(v^i_{n-1})\} = \gcd\{k_i + 2, k_i + 3\} = 1$. Also $f(N(v^i_3)) \supset f(N(v^i_1))$ and $f(N(v^i_{n-2})) \supset f(N(v^i_n))$ and so $\gcd(f(N(v^i_3))) = \gcd(f(N(v^i_{n-2}))) = 1$. It may be verified that each of the remaining vertices also have two neighbors which are labeled with consecutive integers and hence f defines a $(k_i + 1)$-neighborhood-prime labeling on W_i. □

Corollary 2.1. *If G is an arbitrary graph and H is a cycle C_n, then $G \odot H$ is neighborhood-prime graph for all n.*

Proof. We know that the cycle C_n is obtained by joining the two end vertices of the path P_n. Therefore, it may be observed that every $(k_i + 1)$-neighborhood-prime labeling of the subgraph W_i of $G \odot P_n$ is also the $(k_i + 1)$-neighborhood-prime labeling of the subgraph W_i of $G \odot C_n$. The corollary now follows in view of Lemma 2.1 and Theorem 2.1. □

Example 2.1. *A $(k_i + 1)$-neighborhood-prime labeling of the subgraph W_i of $G \odot P_8$ is shown in Figure 1 with $i = 3$ and $k_i = 18$.*

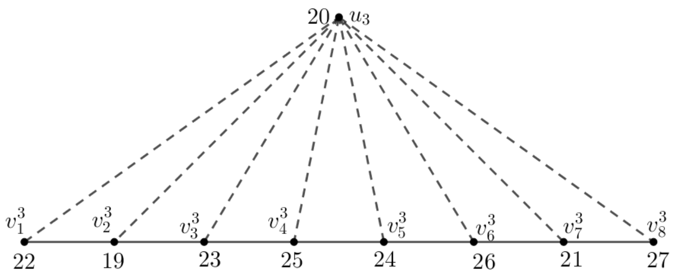

Figure 1

Theorem 2.2. *If G is an arbitrary graph and $H = P_n \; P_n$, then $G \odot H$ is neighborhood-prime graph for all n.*

Proof. We assume that $n \geq 4$ to avoid the relatively easier cases which are left to the reader. We shall define a $k_i + 1$-neighborhood-prime labeling f on each W_i depending on the parity of the number $k_i = (|V(H_i)| + 1)(i - 1)$. Here we consider the vertex set and the edge set of H_i as $V(H_i) = \{v^i_j, w^i_j : j = 1, 2, \ldots, n\}$ and $E(H_i) = \{v^i_j v^i_{j+1}, w^i_j w^i_{j+1} : j = 1, 2, \ldots, n - 1\}$ respectively.

Case 1: k_i is even.

Define

$$f(v^i_2) = k_i+1, \quad f(v^i_{n-1}) = k_i+2, \quad f(u_i) = k_i+3, \quad f(w^i_2) = k_i+4, \quad f(w^i_{n-1}) = k_i+5.$$

If n is odd, then the remaining vertices are labeled as

$$\begin{aligned} f(v^i_{2j-1}) &= k_i + 5 + j; & j &= 1, 2, \ldots, \frac{n+1}{2}, \\ f(v^i_{2j}) &= k_i + \frac{n+11}{2} + (j-1); & j &= 2, 3, \ldots, \frac{n-3}{2}, \end{aligned}$$

$$\begin{aligned} f(w^i_{2j-1}) &= k_i + n + 3 + j; & j &= 1, 2, \ldots, \frac{n+1}{2}, \\ f(w^i_{2j}) &= k_i + \frac{3n+7}{2} + (j-1); & j &= 2, 3, \ldots, \frac{n-3}{2}; \end{aligned}$$

and if n is even, then assign the labels as

$$\begin{aligned} f(v^i_{2j-1}) &= k_i + 5 + j; & j &= 1, 2, \ldots, \frac{n-2}{2}, \\ f(v^i_{2j}) &= k_i + \frac{n+8}{2} + (j-1); & j &= 2, 3, \ldots, \frac{n}{2}, \\ f(w^i_{2j-1}) &= k_i + n + 3 + j; & j &= 1, 2, \ldots, \frac{n-2}{2}, \\ f(w^i_{2j}) &= k_i + \frac{3n+4}{2} + (j-1); & j &= 2, 3, \ldots, \frac{n}{2}. \end{aligned}$$

Thus $\gcd(f(N(v^i_n))) = \gcd\{f(v^i_{n-1}), f(u_i)\} = \gcd\{k_i+2, k_i+3\} = 1$ and $\gcd(f(N(w^i_1))) = \gcd\{f(w^i_2), f(u_i)\} = \gcd\{k_i+4, k_i+3\} = 1$. Also k_i being even, we further observe that $\gcd(f(N(v^i_1))) = \gcd\{f(v^i_2), f(u_i)\} = \gcd\{k_i+1, k_i+3\} = 1$ and $\gcd(f(N(w^i_n))) = \gcd\{f(w^i_{n-1}), f(u_i)\} = \gcd\{k_i+5, k_i+3\} = 1$. But $f(N(v^i_{n-2})) \supset f(N(v^i_n))$, $f(N(w^i_3)) \supset f(N(w^i_1))$, $f(N(v^i_3)) \supset f(N(v^i_1))$ and $f(N(w^i_{n-2})) \supset f(N(w^i_n))$ and so further it is implied that

$$\gcd(f(N(v^i_{n-2}))) = \gcd(f(N(w^i_3))) = \gcd(f(N(v^i_3))) = \gcd(f(N(w^i_{n-2}))) = 1.$$

Each of the remaining vertices possess two neighbors with consecutive labels and so we are done in this case. However, if k_i is odd then we need to modify f as follows.

Case 2: k_i is odd.

We first assume that $k_i \not\equiv 1 \pmod 3$ and define $g : V(W_i) \to \{k_i+1, k_i+2, \ldots, k_i+|V(W_i)|\}$ as

$$\begin{aligned} g(v^i_2) &= k_i + 1, \\ g(v^i_{n-1}) &= k_i + 3, \\ g(u_i) &= k_i + 2, \\ g(w^i_2) &= k_i + 4, \\ g(w^i_{n-1}) &= k_i + 5, \\ g(u) &= f(u); \quad u \in V(W_i) \quad \{u_i, v^i_2, v^i_{n-1}, w^i_2, w^i_{n-1}\}. \end{aligned}$$

We see that $\gcd(g(N(v^i_1))) = \gcd\{g(v^i_2), g(u_i)\} = \gcd\{k_i+1, k_i+2\} = 1$ and $\gcd(g(N(v^i_n))) = \gcd\{g(v^i_{n-1}), g(u_i)\} = \gcd\{k_i+3, k_i+2\} = 1$. Also $\gcd(g(N(w^i_1))) = \gcd\{g(w^i_2), g(u_i)\} = \gcd\{k_i+4, k_i+2\} = 1$ because k_i is odd and $\gcd(g(N(w^i_n))) = \gcd\{g(w^i_{n-1}), g(u_i)\} = \gcd\{k_i+5, k_i+2\} = 1$, because $k_i \not\equiv 1 \pmod 3$. We ask the reader to verify that g is the required labeling in this case also. Also, if $k_i \equiv 1 \pmod 3$, then interchanging the labels of u_i and w^i_2 settles the problem and so we are done. □

The following corollary now follows easily as per the argument of Corollary 2.1

Corollary 2.2. *If G is an arbitrary graph and $H = C_n \quad C_n$, then $G \odot H$ is neighborhood-prime graph for all n.*

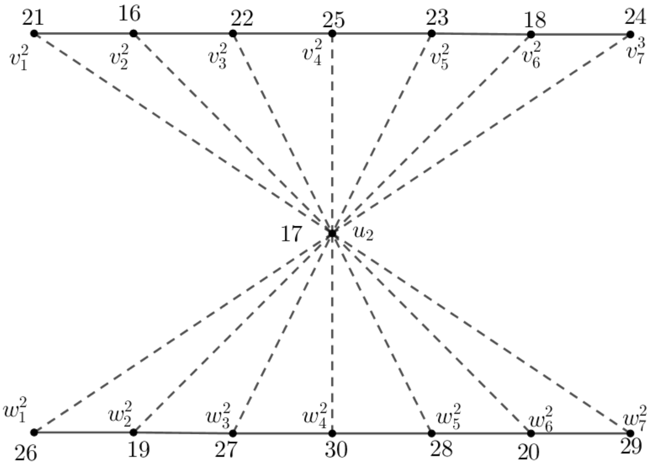

Figure 2

Example 2.2. *A $k_i + 1$-neighborhood-prime labeling of the subgraph W_i of $G \odot (P_7 \quad P_7)$ with $i = 2$ and $k_i = 15$ is shown in Figure 2.*

Let $S_m = K_{1,m}$ be a star graph with m edges. Then a *book graph B_m* (with m-pages) is defined as a graph obtained by taking the Cartesian product of S_m with path P_2. Note that the book graph B_m can also be viewed as m-copies of cycle graph C_4 sharing a single edge. Using this observation the definition of book graph extends naturally to an n-polygonal book graph. An *n-polygonal book $B_{m,n}$* (with m pages) is a graph formed by m copies of an n-polygon (i.e. C_n) sharing a single edge. Next we consider neighborhood-prime labeling of $G \odot H$ in which H is an n-polygonal book with m-pages.

Theorem 2.3. *If G is any graph and H is n-polygonal book $B_{m,n}$, then $G \odot H$ is neighborhood-prime graph for all n.*

Proof. Here we consider the vertex set of H_i as $V(H_i) = \{v^i_0, v^i_1, w^i_{j,l} : j = 1, 2, \dots, m; l = 1, 2, \dots, n-2\}$ and the edge set as $E(H_i) = \{v^i_1 v^i_0, v^i_0 w^i_{j,1}, w^i_{j,l} w^i_{j,l+1}, w^i_{j,n-2} v^i_1 : j = 1, 2, \dots, m; l = 1, 2 \dots, n-3\}$. Then $|V(H_i)| = m(n-2) + 2$ and $|E(H_i)| = m(n-1) + 1$. Define $f : V(W_i) \to \{k_i + 1, k_i + 2, \dots, k_i + |V(W_i)|\}$ by

$$f(u_i) = k_i + 2$$
$$f(v^i_0) = k_i + 1$$
$$f(v^i_1) = k_i + 3$$

For the labeling of $w^i_{j,\ell}$ the following two cases on n has been considered.
Case 1: n-odd.

$$f(w^i_{j,2l-1}) = k_i + 3 + (j-1)(n-2) + l; \qquad l = 1, 2, \dots, \frac{n-1}{2},$$
$$f(w^i_{j,2l}) = k_i + 3 + (j-1)(n-2) + \left(\frac{n-1}{2}\right) + l; \qquad l = 1, 2, \dots, \frac{n-3}{2}.$$

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Case 2: *n*-even.

$$f(w^i_{j,2l-1}) = k_i + 3 + (j-1)(n-2) + l; \qquad l = 1, 2, \dots, \frac{n-2}{2},$$
$$f(w^i_{j,2l}) = k_i + 3 + (j-1)(n-2) + \left(\frac{n-2}{2}\right) + l; \qquad l = 1, 2, \dots, \frac{n-2}{2}.$$

Now

$$\begin{aligned}\gcd(f(N(w_{j,1}))) &= \gcd\{f(u_i), f(v^i_0), f(w^i_{j,2})\}\\ &= \gcd\{k_i + 2, k_i + 1, k_i + (j-1)(n-2) + 5\} = 1\end{aligned}$$

and

$$\begin{aligned}\gcd(f(N(w_{j,n-2}))) &= \gcd\{f(u_i), f(v^i_1), f(w^i_{j,n-3})\}\\ &= \gcd\{k_i + 2, k_i + 3, k_i + (j-1)(n-2) + n\} = 1.\end{aligned}$$

One can check that the other vertices of the type $w^i_{j,L}$ have *two* neighborhood vertices labeled with consecutive integers. Same is true for the vertices v^i_0, v^i_1 and u_i. Thus f is a neighborhood-prime labeling on $G \odot H$. □

Example 2.3. *A neighborhood-prime labeling of the subgraph W_i of $G \odot H$ where H is 5-polygonal book with 4 pages is shown in Figure 3.*

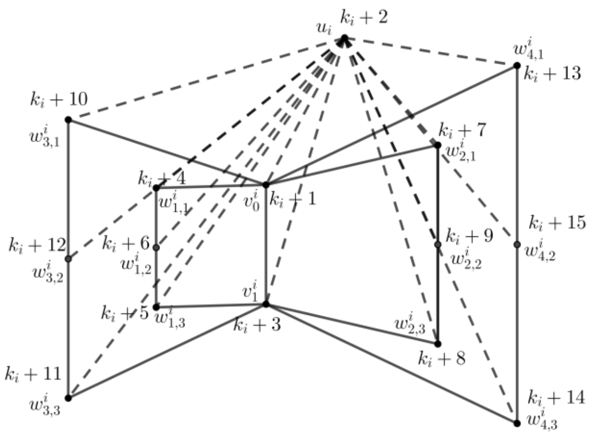

Figure 3

We say that a positive integer $n > 1$ satisfies the property P, if every set of n consecutive integers contains at least one integer which is relatively prime to all the other members of that set. For example, we can say that both 2 and 3 satisfy property P because every set of 2 or 3 consecutive integers contains an odd integer which is relatively prime to the remaining integers in that set. In fact, with a little more effort one can show that all $2 \le n \le 8$ satisy the property P. We provide a quick proof of this in the following lemma.

Lemma 2.2. *The integer n satisfies the property* P *for all* $2 \le n \le 8$.

Proof. Note that we have already discussed the case $n = 2$ and 3. Further, any set of 4 or 5 consecutive integers contain an odd integer t which is not a multiple of 3 and such a t is always relatively prime to the remaining integers of the same set. Now for $n = 6, 7$ and 8 we argue as follows.
Consider a sequence of n consecutive integers $k+1, k+2, \ldots, k+n$ for an arbitrary positive integer k. If $n = 6$, then there exists a positive odd integer $t \not\equiv 0 \pmod 3$ such that $k+1 < t < k+6$. Such a t is relatively prime to the remaining 5 integers of the sequence since it is not a multiple of 3 and its distance from these numbers is at the most 4. Finally if $n = 7$ or 8 then observe that there is a positive odd integer t such that $t \not\equiv 0 \pmod 3$, $t \not\equiv 0 \pmod 5$ and $k + 1 < t < k + 8$. The number t satisfies our needs because it is not a multiple of 3 or 5 and also its distance from rest of the numbers in the corresponding sequence is no more than 6. □

Theorem 2.4. *Let H be any graph of order n and whose total domination number is $d-1$. If the number d satisfies the property P, then $G\odot H$ is neighborhood-prime graph for all G.*

Proof. Let W_i be the subgraph of $G \odot H$ induced by the vertices of the set $\{u_i, v^i_j : j = 1, 2 \ldots, n\}$. In order to prove that $G \odot H$ is neighborhood-prime, it is enough to label the vertices of the set $\{u_i, v^i_j : j = 1, 2 \ldots, n\}$ with the integers $k_i + 1, k_i + 2, \ldots, k_i + n + 1$ such that $\gcd(f(N(u))) = 1$ for all $u \in W_i$, where $k_i = (n + 1)(i - 1)$. For this, we consider a total dominating set S_i of H_i with $d - 1$ vertices. Since the number d satisfies the property P, there exists an integer $t_i \in [k_i + 1, k_i + d]$ which is relatively prime to all the other integers in that interval. We label the vertex u_i with t_i and the vertices of the set S_i by the (distinct) integers $k_i + 1, k_i + 2, \ldots, k_i + d$ which are different from t_i. The remaining vertices of W_i can be randomly labeled with the integers $k_i + d + 1, k_i + d + 2, \ldots, k_i + n + 1$. Since the neighborhood set of every vertex in H_i contains a vertex of the set S_i along with u_i and further, the neighborhood set of u_i contains vertices with consecutive labels, it follows that $G \odot H$ is neighborhood-prime. □

In view of Lemma 2.2 and Theorem 2.4, we can conclude that if the total domination number of a graph H is at most 7, then the graph $G \odot H$ is neighborhood-prime for all G. In particular, we can say that the corona product of any arbitrary graph G with a wheel graph W_n, a complete graph K_n or a complete bipartite graph $K_{m,n}$ (with $m \leq 7$) is neighborhood-prime.

Theorem 2.5. *If G is an arbitrary graph and H is a comb graph $P_n \odot K_1$ such that $|V(G \odot H)| \geq 2^{|V(G)|}$, then $G \odot H$ is neighborhood-prime.*

Proof. We consider $V(H_i) = \{v^i_j, x^i_j : j = 1, 2, \ldots, n\}$ and $E(H_i) = \{v^i_j v^i_{j+1}, v^i_j x^i_j, v^i_n x^i_n : j = 1, 2, \ldots, n - 1\}$. Then $|V(H_i)| = 2n$ and $|E(H_i)| = 2n - 1$. Also if $p = |V(G)|$, then $V(G\odot H) = V(G) \cup (\bigcup_{i=1}^{p} V(H_i))$ and $E(G\odot H) = E(G) \cup (\bigcup_{i=1}^{p} E(H_i)) \cup \{u_i u : i = 1, 2, \ldots, p; u \in V(H_i)\}$. So $|V(G \odot H)| = p(|V(H_i)| + 1) = p(2n + 1)$ and $|E(G \odot H)| = |E(G)| + p(|E(H_i)| + |V(H_i)|) = |E(G)| + p(4n - 1)$. Using the fact that $|V(G \odot H)| \geq 2^{|V(G)|}$, we define $f : V(G \odot H) \to \{1, 2, \ldots, |V(G \odot H)|\}$ by

$$f(u_i) = 2^i,$$
$$f(v^i_j) = \begin{cases} 2j - 1; & i = 1, \\ (2n - 1) + 2n(i - 2) + 2j; & \text{otherwise}, \end{cases}$$

whereas the remaining vertices of $V(G \odot H)$ are labelled randomly from the set $\{1, 2, \ldots, |V(G \odot H)|\} \setminus \{f(u_i), f(v^i_j) : j = 1, 2, \ldots, n; i = 1, 2, \ldots p\}$. Notice that the definition of f assures that if v is either v^i_j or x^i_j, then one of its neighbor always has an odd label. Therefore $\gcd(f(N(v))) = 1$ because $\{f(N(v))\}$ also contains $f(u_i) = 2^i$ whose gcd with an odd number is always 1. Also $\gcd(f(N(u_i))) = 1$ because $N(u_i)$ contains v^i_j's which are labelled by consecutive odd integers. Consequently, f is a neighborhood-prime labeling on $G \odot H$. □

The definition of f is illustrated for the subgraph W_4 of $G \odot (P_5 \odot K_1)$ in Figure 4.

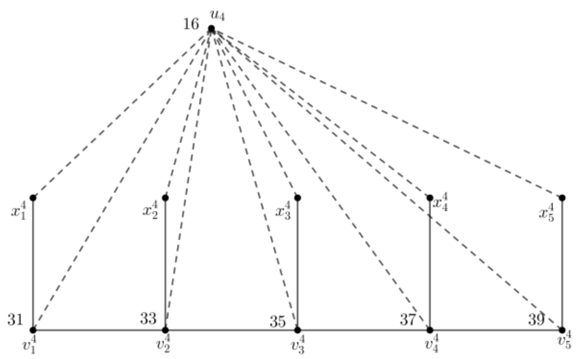

Figure 4

Since a crown graph $C_n \odot K_1$ can simply be obtained by adding a single edge to the comb graph $P_n \odot K_1$, the following corollary is now immediate.

Corollary 2.3. *If G is an arbitrary graph and H is a crown graph such that $|V(G \odot H)| \geq 2^{|V(G)|}$, then $G \odot H$ is neighborhood-prime.*

3 Conclusion

The present study is focused on neighborhood-prime labeling of graphs obtained by taking corona product of an arbitrary graph with graphs of specific families. This type of investigation can be carried out for the other graph families also. In particular, it will be interesting and challenging to study the corona product of an arbitrary graph with certain families of tree graphs.

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